My purpose is to state necessary truths about numbers, not sufficient truths.

Dick,

1. It was good seeing you the other day. Below are two "streams of consciousness" on numbers. Let me know what you think.

Best,

Jack

First:

In discussing Aquinas's a theory of numbers Thomists have a shockingly avoided what are known as transcendental numbers, for example, the number of persons in the Trinity. It is these numbers, rather than the numbers that result from the division of all continuum, that are what modern mathematics is talking about.

But in what follows I am going to have to use the word transcendental in two different senses. When speaking of transcendental numbers, we mean numbers that transcend the sphere of the material, numbers that can be shared by material beings and immaterial beings, just as other transcendental properties can be. But the analysis that follows also depends on comparing the concept of number to what are known in tradition as the transcendentals. With that warning, I hope I can make things clear for the reader.

Consider the Trinity. Is the quantity 3 an accident of the Trinity, an accidental feature inhering in the Trinity? If not, is number merely a being of reason, a logical construct of some sort.? No, the number of persons in the Trinity is a reality; that is, the fact that there are three persons in the Trinity rather than two or four it is a reality. Likewise, if God had created only three creatures, say three angels, the fact that

there were three creatures rather than two or four would be a reality. Yet, the No. 3 would not be an accident of any of the 3 Angels. So what kind of concept is number?

We know that number is associated with the extension of a universal concept. In order for there to be three, there must be three of something: three oranges, three apples, three persons, or whatever. That is why the concept of number can appear to be a being of reason. For universality and extension are logical relations and therefore beings of reason. But the fact that a concept includes a being of reason does not imply that the objective concept as a whole is the objective concept of a being of reason. In fact, it means the opposite. The whole point of the doctrine of the transcendentals, that is, of the concepts convertible with being, the one the good the true etc., is that the addition of a being of reason to the concept of being. The whole point of the doctrine of the transcendentals is that the first concept, the concept of being. The whole point of the doctrine of the transcendentals is that the true is not a being of reason but is convertible with, that is, identical with, being. Not identical as an objective concept, but identical as to the reality objectified by each of these objective concepts.

The reason for this identity is precisely that the concept of the true adds only a being of reason to the concept of being and therefore does not add any reality to the concept of being. Something similar must be true of the concept of number.

To the reality all of the persons in the Trinity the No. 3 adds the concept of falling under the extension of the concept of person. It objectifies the realities that fall under the extension of that concept precisely as falling under the extension of a concept. But what falls under the extension of a concept is not a being of reason, or need not be, it can be a reality and in the first instances must be a reality. To the reality that exists when the Trinity exists, or the reality that exists when only three creatures have been created, the No. 3 adds the being of reason of a falling under the extension of the concept of person or the concept of creature, respectively, that is, of being a term of the relation of universality that characterizes the objective concept of person or creature. But it adds more. It adds the concept of falling under the extension of a universal concept together with the other things that fall under the extension. Or, in common with the other things that are terms of the relation of universality.

To the reality that exists when the Trinity exists or three creatures exist, the concept of number adds the

being of reason of so many falling under the extension of a concept together, the concept of so many falling under a universal concept in common. So the existence of three or the existence of 4 is not a being of reason it is identical with the existence of the realities being objectified. But it objectifies those realities in a certain way, just as the concept of true objectifies being in one way in the concept of good objectifies being in another way. Note that I have not spoken about a set of realities. The concept of set comes later and the concept of set it is the concept of a being of reason. The No. 3 objectifies the persons that exists in the Trinity and the creatures that exist, if only three creatures have been created, by a specific way of falling under the extension of a concept together. Here specific refers to a species as opposed to an individual, on the one hand, or a genus, on the other. The persons of the Trinity fall under the concept of person together or in common, in the same way, or in anyway that is the same as a way that the 3 creatures fall under the concept of creature together.

But note also that number can apply to beings of reason as well as to real beings. For beings of reason can also fall under universal concepts; rather, they do also fall under universal concepts. And the mathematician looks at numbers in abstraction from whether the No. 3 is identical with a kind of reality that exists when for example the Trinity exists or a triangle exists. And so for the purposes of the mathematician, she can construct beings of reason and consider them numbers on a par with the whole numbers, even though a being of reason like zero, negative numbers, the square root of negative one, etc. are not and cannot be ways in which realities fall under the extension of concepts in common with other realities.

The following may be a little repetitious. Assume that only these realities exists, Tom, Dick and Harry. We want to say that when these realities exists, multiplicity exists. And we also want to say that when these realities exists, multiplicity is not something that exists over and above what exists when Tom Dick and Harry exist. So multiplicity is a reality, but it is not a reality distinct from what exists when Tom, Dick and Harry exist. We can also say that when Tom, Dick and Harry exist, threeness exists.

Threeness is a species of multiplicity. Whenever multiplicity exists, some species of multiplicity must exist.

Along the same lines, but from a different point of view, another thing we want to say is that the above

description of number does not imply that multiplicity comes into existence only when universal concepts with extension come into existence. To say that when Tom Dick and Harry exist, threeness also exists is not to say that there also must exist a human knower with the universal concept, say, of person. When Tom, Dick and Harry exist, the reality that exists, the extramental reality that exists, is such that it is capable of truthfully being objectified by the No. 3, because, the concept of person includes Tom, Dick and Harry, and only Tom, Dick and Harry, in its extension. But that which is capable of being so objectified truthfully exists prior to any such objectification. What exists prior to the objectification are potential terms of the concept of person's extension, where the potentiality does not refer to what exists prior to the objectification, but to the potentiality of the objectification, the potentiality of the objectification's occurring.

In other words, what exists prior to the objectification is multiplicity, in particular, threeness. The multiplicity consists of entities capable of becoming objects of the concept person. We can say that the No. 3 exists prior to the concept of person in the sense of numbered number, that is, of what is counted by the word three, the word three being numbering number. So to say that multiplicity exists is to say that reality is such that it is capable of being truthfully objectified by numbering number, where numbering number means that reality plus the relation of reason of there being terms of the extension of a concept together.

When entities, Tom, Dick, and Harry exist, threeness exists. When entities Tom, Dick, Harry and Sam exist, fourness exists.

Second:

Sets are beings of reason. They belong to the third order of abstraction, because that is where number, even in the classical sense, belongs. We, including Simon, have ignored completely Aquinas's treatment of "transcendental" number in the treatise on the Trinity. Surely the quantity of persons in the Trinity is as real as any other quantity. Division of a material continuum may be necessary to supply us with the examples from which our psychological process of learning about numbers must begin. But the truth of arithmetical sentences does not require "common intelligible matter."

Then what is number? Assume God created only Larry, Moe and Curly. Did He also create threeness? If so what is it? An accident? If so, an accident of whom? A substance? If so, that would be Pythagoreanism with a vengeance. In fact, there is another way out within Thomism, though not one ever applied to this problem before, to my limited historical knowledge.

Numbers are in important respects like the True, the Good, the Beautiful, etc. (Call them here the "convertibles." Though I am comparing "transcendental" numbers to them, I am not comparing them with respect to the meaning of the word "transcendental"; in fact, the meanings are different in each case. Three is not convertible with being, but it is "transcendental" in the sense of being truthfully asserted of immaterial being.) The true, the good, etc. are 100-percent realities; that's why they are convertible with being. The fact that to objectify being in these ways we must make use of relation of reason does not mean that what is so objectified is in anyway not real. It means the opposite. Because the only difference between the true and what exists is a relation of reason added to what exists, there is by hypothesis no real difference between them; what is objectified by "true" is by hypothesis identical with the reality objectified by "being."

Likewise, threeness, or the number three, adds to the reality of Larry, Moe and Curly only the relation of reason of being the extension of such objective concepts as "created beings," "finite beings," "caused beings," "human," "animal," "rational," "actors," etc. So when God created L, M and C, He did indeed create threeness, but He did not create any reality in addition to L, M and C. Threeness is a reality (or a characteristic of reality, and for there to be three beings, not two or four, is something real) but is neither a substance nor an accident. It is what exists objectified by means of a certain kind of being of reason. Likewise, when God created L, He created something true, good etc. But he did not created them as realities in addition to L. (Pace Benacerraf, I'd like to see this argued in an article entitled "What Numbers MUST Be.")

The philosopher can recognize this as the psychological source of the concept of set, which of course goes far beyond the kind of set that is its psychological source.