Does Everything Follow from Contradiction? Yes and No

An Epistemological Analysis of Modern Logic and Traditional Logic

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The issue I am raising belongs to the epistemology of logic rather than to logic proper: how do we know that a computational procedure establishing valid logical consequence is correct according to the rules of the computational method? Except where more specificity is relevant I will, for the sake of brevity, use 'rules' as a generic term covering any means we can employ as bases for establishing validity computationally, such as axioms, laws, inference principles, definitions, etc., and for any computational device, including natural deduction, matrices, trees, diagrams or whatever. How do we know that in a decision procedure or proof we have correctly applied the rules of the method?

By a 'computational' method I mean one in which we follow rules that describe empirically verifiable steps, rules such that we can empirically verify statements like 'This step follows rule such and such', or 'This step is an instance of rule such and such'. For example, a rule of bridge states that if a player has a card in the suit that was led, the player must play a card in that suit. Presumably, bridge players are capable of empirically verifying whether a card led is in a particular suit and whether they have a card in that suit. So they are capable of recognizing that a particular situation is one that this rule is designed to cover, and therefore of knowing the empirically verifiable action that the rule calls for in this situation.

Knowledge that a computational step is justified or required by a rule may not seem to present much of a mystery. But I will argue that an analysis of how we know that just one step in a computational process is correct requires a positing of two distinct epistemic types: (1) knowledge that certain statements are necessarily true and (2) knowledge of certain contingent truths (or at least truths that need not be epistemically necessary). As the bridge example indicates, we wouldn't call just any rule-following procedure computational. As further specificity about what 'computational' methods are is required in particular contexts, I will supply it, but sufficiently only for the purposes of this study[1], where I intend to show that, whatever else may be true of computational methods, knowledge of computational correctness presupposes knowledge of necessary truth *gained non-computationally*. So I only need to characterize 'computational' methods sufficiently to show why the knowledge of necessary truth their use presupposes is NOT gained by using those methods.

Recognizing that a step correctly instantiates a rule is not the same as recognizing that the rule or any formula we arrive at by means of it expresses a necessary truth or is connected in some other epistemically interesting way to necessary truth. To this extent, those who hold that modern logic does not require the existence or knowledge of necessary truths are right. But rules do not apply themselves. Knowledge that a computational step correctly applies a rule involves making an inference whose validity we recognize, and we can recognize the validity of an inference if and only if we recognize the inference as an instance of a necessary truth about inferential validity. So we need knowledge of necessary truths of the kind we happen to call 'logical' to account for our knowledge of the correctness of any computational procedure. And it does not matter whether the computational procedure we recognize to be correct is at the metatheorem or theorem level, or whether the procedure bears on formulas that are schemata or sentences. The epistemic questions I am asking are the same for any use of computational methods for logic.

In addition to the recognition that some statements are true on penalty of contradiction, knowledge of computational correctness requires the recognition that these and other statements about a computational process are bivalently true. I will call the necessary and bivalent truths about inferential validity, knowledge of which is presupposed to the grasp of computational correctness, principles of 'standard' logic Knowledge of necessary and bivalent truths about valid inference is sought in both traditional logic and classical modern logic (so 'standard logic' will not be synonymous with classical modern logic). But knowledge of such truths is a precondition for recognizing the computational correctness of procedures in logics that are multivalued, paraconsistent, dialethic, or any other form of nonstandard logic.

Of course, we can express the rules to be followed in computations by means of directions that are neither true nor false, like 'Delete a true formula as a component of a conjunction'. Being neither true nor false, such rules telling us steps allowed in computational procedures do not express something logically necessary. But in classical modern logic we construct our computational methods by choosing rules on the basis of our knowledge of the necessity of truths like 'Deleting a truth as a component of a conjunction preserves truth-value'.

The 'computational' methods I have in mind are, more specifically, those using formal languages and formal definitions of logical consequence, whether semantic or proof-theoretic. Epistemically, the most important motive for using computational methods is to be able to know the contingent truths that each of a series of steps has correctly applied a rule or rules. To achieve that end we use rules whose correct application is empirically verifiable. To achieve that end for logic, we use empirically verifiable rules about marks and arrangements of marks.

Further, the rules of classical modern logic so determine the use of otherwise undefined marks that we know the marks can be empty placeholders for names, predicates, pronouns, and/or declarative sentences. To these are added other rules for arrangements using the preceding marks together with other marks for 'logical' operators and quantifiers. But we could so gerrymander the latter rules that we know both that the correct application of the latter rules yields certain arrangements of marks and not others and that, as the bridge example illustrates, the rules have no interesting connection, from the point of view of the epistemic conditions of the use of computational methods for logic, with any precomputationally known necessary truth. Such gerrymandered rules, however, would not allow us to use computational methods to verify valid logical consequence. Instead, therefore, classical modern logic so chooses the latter rules that we can know the following about at least some of them: If we replace the placeholders in them according to the former rules, the interpreted formula expresses a valid inference, or a necessarily true inference principle, on penalty of denying something we precomputationally know to be necessarily true. In this way we can verify logical validity by empirically verifying that a process arriving at a particular arrangement of marks is computationally correct according to rules that must preserve truth.

I will argue, for example, that we can precomputationally know necessary truths such as

(a) If ‘The road is wet, it is slippery’ and ‘The road is wet’ are true, ‘It is slippery’ is true.

And we can know from the way we construct the rules of a formal language that (a) would be a correct interpretation of a rule like

(a’) ((p -> q) & p) -> q.

But since the knowledge of necessary truth required for grasping the validity of a computational inference not only is not but cannot be achieved computationally, *no computational method can provide an entirely accurate model of how we precomputationally achieve knowledge of necessary logical truths* (which does not lessen the importance of those methods, to be described in a moment, one bit). So the fact that the grasp of inferential validity presupposed by the knowledge of computational correctness is the grasp of a necessary truth that an inference form is valid does not imply that, at the computational level, a proof-theoretic definition of logical consequence is more useful, more powerful or more in accord than a semantic definition with the ‘correct’ concept of logical consequence at the epistemic level, the level of the grasp of the validity of an inference form that is presupposed by the grasp that a procedure conforms to computational rules. Such questions mix apples and oranges.[2] The rock bottom metaphysical explanation of what causes truths to be necessary and/or epistemic explanation of what causes our knowledge that a truth is necessary might resemble a semantic computational approach to logical consequence more, a proof-theoretic one more, not resemble one more than the other, or not resemble either. But the existence of some resemblance would be evidence for no more than that, just a resemblance.

The concept of necessary truth that will be pertinent to this inquiry is that of a statement whose denial is contradictory, a statement whose denial either is or implies a contradiction. So the fact that the grasp of inferential validity is the grasp of a necessary truth does not imply that what valid inferences preserve is modal truth. What is grasped is that the denial of a statement is contradictory. Computational methods for modal logic presuppose a non-computational grasp that the opposite of statements about valid inference are contradictory just as any computational method does.

I will not define a necessary truth as true in all possible worlds because the computational use of possible world semantics is an instance of the kind of knowledge whose epistemic pre-conditions I am investigating. Knowledge of the correctness of a computational step in possible world semantics requires a grasp of necessary truth gained non-computationally just as much as any knowledge of computational correctness does. To understand our knowledge of inferential validity we need an account of necessary truth that is epistemically deeper than the possible worlds account. I will offer such an account.

To establish the distinction between the two epistemic types needed for knowing computational correctness, I begin with a refutation of procedures claiming to show that everything follows from contradiction. We can indeed reach any *q* from any (*p*~*p*) using rules of inference, such as disjunctive syllogism and material implication, whose validity seems necessarily true. But we can also know that such principles no longer have logical necessity if negating *p* no longer does the logical work of taking away *p*. Thus, knowledge that a formula results from a correct application of computational rules is one thing; knowledge that rules and what follows from them express necessary truths is another. I will call the first kind of knowledge computational knowledge. Since the science of logic has traditionally (and accurately) been thought to know necessary truths, I will call the second kind of knowledge logical knowledge. The difference between contingent and necessary truth makes the difference between them as objects of knowledge epistemically important, even though the computational method of grasping contingent truth includes a grasp of necessary truth.

I intend to show that computational knowledge is not only different from logical knowledge but relies on logical knowledge, logical knowledge that, therefore, cannot be achieved by the use of computational methods. It does not rely on logical knowledge as knowledge of a conclusion relies on knowledge of the premises. It relies on logical knowledge that way any science relies on recognizing that its inferences are valid. To show that, I will then argue that recognizing computationally that a rule is correctly applied requires recognizing the validity of an inference; then I will argue that recognizing the validity of an inference requires recognizing that a statement about inferential validity is necessarily true.

As in all sciences, reasoning in traditional logic and classical modern logic presupposes an implicit grasp of necessary logical truths that is implicit. Traditional and classical modern logic both seek to make our grasp of inferential validity explicit. (The contrast between implicit and explicit knowledge will also be explained sufficiently for the purposes of this discussion.) But modern logic does this by means of an incomparably more powerful tool, computational methods. Since knowledge of computational correctness and knowledge of the necessary truth of inferential validity are distinct epistemic types, however, the relation of computational knowledge to the explicit logical knowledge that traditional and classical modern logic seek is like the relation of mathematics to physics, which are also distinct epistemic types: Computational knowledge is not the same as logical knowledge, but just as mathematics is an indispensable and incomparably powerful tool for physics, computational knowledge is an indispensable and incomparably powerful tool for advancing our explicit logical knowledge. To want to go back to traditional methods of doing logic where computational methods, either now or in the future, can succeed would be like wanting to go back to pre-Galilean, or even pre-Archimedean, or even pre-Pythagorean physics because we are able to know some physical truths without mathematics.

To show that knowledge of inferential validity requires recognizing that some statements of logic are necessarily true, I will use overlooked implications of Lewis Carroll's Achilles-Tortoise paradox. And a refutation of Quine's critique of the 'analytic' will later show that grasping the necessary does not require occult mental acts but some of the same acts, if any, required for grasping contingent empirical truths; whatever it may take to grasp contingent truths, it takes no more to grasp necessary truths. In fact, knowledge of logical necessity bespeaks limited mental powers, not extraordinary ones.

Since we can be aware of logical necessity, the technical success of nonstandard logics at the computational level does not justify pragmatism in the epistemology of logic.[3] By the technical success of these systems I mean two things. First, we can computationally verify that their procedures are correct according to their rules. Second, we can computationally verify that their rules, though not all reflective of necessary truth, do cover all the possible cases they are designed to cover (a type of verification, not confined to completeness proofs, I will discuss in Section 4). It has always been recognized that we must use logic to do logic. I am just adding that, because of the epistemic preconditions of computational verification, the logic we must use to do any kind of logic computationally, including many-valued, paraconsistent, and dialethic logics, is logic that is standard in the sense that it does not permit contradiction and is bivalent. Unless we can know inference principles that are necessarily and bivalently true, we could not verify the truth even of statements like 'This nonstandard system succeeds computationally' or 'This step is correct according to this nonstandard rule'.

To put this epistemic inquiry in context, consider this statement of Graeme Forbes:

> A distinction between propositions (or statements, or sentential contexts) which are de dicto and propositions (and so on) which are de re originates in medieval philosophy. But only contemporary modal logic affords the tools for a precise characterization of this distinction, although it must be granted that the distinction remains a puzzle in *epistemic* contexts. (Emphasis in original.)[4]

Also as Hilary Putnam said about Tarski's treatment of truth:

> The critics (of William James account of truth) also claim the problem of giving us a satisfactory account of truth was solved in this century by the work of the great logi-

> cian Alfred Tarski. I myself believe that Tarski's great technical contribution notwithstanding, his work does *nothing* to explicate the notion of truth. (Emphasis in original.)[5]

Elsewhere, Putnam backs this up by demonstrating that Tarski's technical achievement tells us nothing about the philosophical problems concerning truth.[6] Similarly, I am arguing that the technical success of nonstandard computational logics tells philosophers nothing about the epistemic status of consistency, bivalence, or necessary truth.

Concerning the latter, Kripke said:

> I do not think of 'possible worlds' as . . . in any philosophically significant sense . . . uncovering the ultimate nature . . . of modal operators, propositions, etc., or as 'explicating' them. . . . Philosophically, we by no means need to assume that one type of discourse ('ordinary', 'directly expressed modal locutions' and 'the notion of a "possible world" at a much greater, and subsequent, level of abstraction') is 'prior to' the other, *independently of the purposes at hand* (my emphasis; see n. 2). The main . . . motivation for the 'possible worlds analysis' . . . was that it enabled modal logic to be treated by the . . . set theoretic techniques of model theory.[7]

So the main motivation was to use a particular *tool* for doing modal logic.[8] While that tool makes 'certain concepts clear' (ibid.), Kripke believes it does not explicate modality in any philosophically significant sense. My purpose at hand is to try to show why: an epistemic precondition for the success of computational possible world semantics is the noncomputational grasp that the opposites of some statements are contradictory.

Putnam also has demonstrated that the idea of a 'sound' (logically valid with true premises) proof cannot be formalized in a system that we could recognize as sound computationally.[9] Again, I am offering a reason why: Recognition of the correctness of a computational process does not cause our recognition of the validity of anything, or the truth of anything other than that correctness itself, as the bridge example illustrates; rather recognition of computational correctness is caused by recognition of the validity of an inference, which in turn is caused by the implicit *knowledge* of the self-evidently necessary truth of an inference principle. The computational clarity and rigor of all modern logic presupposes the occurrence of consistent and bivalent epistemic necessity.

If this epistemic thesis is correct, it *should* be considered very minor and unimportant by whose chief interest is doing modern logic for its own sake, or for the sake of its power and fecundity; for there just is no way other to get as far in logic as we do by using computational methods. But it is of more than minor importance for those of us interested in the epistemic pre-conditions for knowing what we are doing when we are doing modern logic. It is also more than a minor point for those who want to draw philosophical conclusions from what modern logic can do. Now that we have nonstandard logics, the impression can be that they eliminate the epistemic need for necessary truth, consistency and bivalence. Just as Tarski's great technical achievement tells philosophers nothing about the notion of truth, the technical success of nonstandard logics tells philosophers nothing about the epistemic (or metaphysical) status of consistency, bivalence, or necessary truth.

The problem, of course, is that those whose teaching in one class concerns epistemic issues about modern logic are very often the same people who were teaching modern logic itself in the pervious class. It is very difficult to shift from the point of view where it is hardly worth noting that you are depending on implicit knowledge of necessary logical truth, since you couldn't even begin to do what you are doing otherwise, to a point of view where such issues create thorny (to say the least) problems to be solved.

What has been going on as we have looked to post-Fregean computational logic for solutions to or escapes from traditional problems about knowledge and truth has been going on in philosophy for almost 2500 years. We apply a method that ultimately turns out to be ineffectual because: (1) We are justifiably dazzled by the genuine power the method possesses in some field other than philosophy. And (2) we correctly perceive some resemblance between a question the method successfully deals with in its own field and some hitherto unsolved philosophical problem (and for sociological philosophic purposes[10] don't all philosophical problems remain unsolved?). The combination of dazzling intellectual power and perceived resemblance causes us to focus on applying the method to the problem while unintentionally ignoring the inconvenient details about the problem that would show us that the method is inappropriate for it. But we needn't feel shame if we find ourselves making the same mistake(s), though in new disquises, that the greatest philosophic minds have been making for 2500 years.

There may be a temptation to think 'But if we don't have modern logic to go on in trying to answer our philosophical questions, what do we have to go on?' What we have to 'go on' are the same preconditions that modern logic must rely on. That may not be much, but it had better be enough.

1.

The technical success of non-standard logics can cast doubt on whether at least some of logic's formulas express necessary truths (or are epistemically connected with necessary truths in some other important way). For example, some philosophers and logicians are bothered by the apparent soundness of procedures, in traditional and classical modern logic, showing that everything follows from contradiction, *ex contradictione quodlibet* (ECQ). Logicians and philosophers of logic interested in the 'entailment' of a specific conclusion by a specific finite set of premises want to avoid the problem that any contradiction entails any statement whatsoever. Paraconsistent and relevance logics prevent ECQ from holding by treating other inference principles, disjunctive syllogism in particular, traditionally considered necessarily true as if they were not.

But from an epistemic point of view, we don't have to worry about ECQ. We don't need nonstandard logics to avoid the problems that ECQ seems to get us into. For we are as capable of recognizing that attempts to justify ECQ do not and cannot succeed as we are capable of recognizing, for the sake of determining inferential validity, that a computational proof does succeed. Methods of showing that $(p^{-}p)$ -> q is valid do illustrate that we can recognize that each step in the process is justified by a rule or a rule together with preceding steps that satisfy rules. From that viewpoint the answer to the question, 'Is everything true, if contradiction is', can be yes. But from the viewpoint of our ability to recognize a rule's association with logically necessary truth, the answer must be no.[11] The reason is that permitting contradiction prevents there being any necessary truth for the rules the procedure uses to be associated with. (Do not confuse ECQ with the fact that a truth-functionally inconsistent antecedent implies everything. Inconsistent antecedents make their conditionals valid because the antecedents resolve to *falsehood*; ECQ is the claim that if both parts of a contradiction are simultaneously *true*, and so the compound sentence *p~p* is true, everything is true.)

A traditional proof that ~(*p~p*) -> *q*, where *q* can be anything, is valid is

(1) *p~p*	Premise
(2) *p*	1, Law of simplification
(3) *p* v *q*	2, Law of addition
(4) ~*p*	1, Law of simplification
(5) *q*	3, 4, Law of disjunctive syllogism
(6) (*p~p)* -> *q*	1, 5, Conditional proof

But if we allow contradiction, the law of disjunctive syllogism (step 5) cannot do the logical work that the argument relies on. We can call disjunctive syllogism a 'law of subtraction'. By denying that *p* is true, we subtract it from *p* or *q*, and *q* is the remainder. But how do we deny the truth of *p*? By the truth of ~*p*? Normally, that is how we accomplish denying *p;* for that is the job we normally give 'not' and other negation signs, the job of denying, subtracting, canceling, deleting, taking away. But the truth of ~*p* does not mean that *p* is false, if *p~p* is also true. If ~*p* does not make *p* false*, p* or *q* does not license us to conclude q from ~*p*. (Or more technically, disjunctive syllogism is no longer 'truth-functionally valid' — or true on every bivalent evaluation, or true under every possible assignment of T or F to atomic statements, etc., whatever you consider the most appropriate technical concept; for it is not true in the case where *p~p* is true. In addition to technical terms, I will not only continue to also use non-technical terms, but will use them in preference where possible; for the issue is the epistemic status of our technical methods, especially the epistemic conditions *presupposed* by the technical clarity and rigor of computational methods. Since I am explaining the epistemic conditions required for achieving the clarity and rigor associated with computational technical vocabulary, the explanation cannot presuppose that clarity and rigor except as the fact given to be explained, not as the means of explanation. The epistemic conditions that give us the ability to design and use computational methods are causally *prior to* the existence of their technical vocabularies, and so to the clarity and rigor with which we endow those vocabularies. Still, any such explanation may have to introduce technical terms of another kind, philosophical terms, endowed with whatever clarity and rigor is called for by the purpose at hand. And as with technical terms of any kind, their introduction must rely, ultimately, on the use of nontechnical vocabulary.)

Epistemically, disjunctive syllogism's (or *modus tollendo ponens'*s) function depends on excluding contradiction. The dependence is not that of a conclusion on a premise; rather, disjunctive syllogism's function depends on a use that negation signs sometimes — and in fact, normally — have. The logical work disjunctive syllogism does depends on the value that we happen to, but need not, use 'not' and '-' for, the relation *other-than*.[12] The existence of multiple kinds of negation is not an issue here. The contradictions about which ECQ is supposedly demonstrated are the simultaneous affirmation and denial (or positing and taking away, or whatever) of the same thing by means of a negation sign used as we happen to ordinarily use it in two-valued contexts; so that is the kind of 'negation' in play when we are talking about ECQ. Traditional principles of noncontradiction – whether for sentences or predicates, schema or schematic letters, in an object language or a metalanguage – just express the job we normally give to negation signs: Sentence or schema $\sim p$ is a denial of p, and vice versa; predicate 'F' cancels 'non-F', and vice versa.

But in any proof of ECQ, the sign by which we indicate that one premise is the contradictory of another must have recognizably the same use in that premise as in any rule (or axiom or whatever, depending on how the computational method is structured), such as disjunctive syllogism or material implication, that allegedly makes any q a logical consequence of a contradiction. If not, we will recognizably commit a fallacy of equivocation. So however the sign indicating contradiction operates in the rule to justify our concluding to q, the sign must operate in the same way in one of the contradictory premises. If the rule does not use that sign or an equivalent sign, or if it uses the same sign but in a different way, the contradictory premise and the rule are irrelevant to each other. This is another reason why the existence of multiple kinds of negation is not an issue, as long as the use negation signs have in contradictions is the same as whatever use they have in disjunctive syllogism. Contradictory premises are irrelevant to deriving q from '¯p via disjunctive syllogism if the negation signs do not have the same use. All we would have accomplished would be to change the subject, as Quine said about attempts to deny principles of non-contradiction.[13] And the subject(s) originally under discussion are contradiction, and so negation, as they are understood in the context of standard bivalent logic, whether traditional or classical modern.

That disjunctive syllogism's work 'depends on' the use we normally give negation signs does not mean that it depends on contingent lexicological facts such as that we happen to use noises like 'not' and shapes like '-' as signs for negation. Disjunctive syllogism depends on that which we happen to, but need not, use 'not' and '-' for. But disjunctive syllogism does not depend on the relation between 'not' or '-' and standard negation being what it happens to be; disjunctive syllogism depends on standard negation being what it happens to be.

The distinction between the lexicological sense of 'use' and its sense in 'disjunctive syllogism depends on the use that we normally give negation signs', does not require some special mental state, different from whatever mental states (if any) may explain our understanding of how shapes, noises or actions are used as signs, to account for our understanding of what negation is. Knowledge of how we use negation signs includes a knowledge of what negation is. And I will argue that any mental states required to explain our knowledge that contradictions cannot be true must be of the same kind as whatever mental states may be required to explain our empirical knowledge that, for example, the letter-token, *T*, is *not* the letter-token, *T*. Also, someone could confuse the way we use 'is not' with the way we use 'is'. If they were to say 'A is not A', we would not have to conclude that they were employing a nonstandard logic. We could have *behavioral* evidence that the way they were using 'is not' is the way the rest of us use 'is'. That behavioral evidence would tell us that they were lexicologically mistaken about the uses of 'is' and 'is not', not logically mistaken about the work that standard negation does.[14]

No inference that requires negating something in the standard way is valid if the truth of ~*p* does not succeed in making *p* false; so no valid inference can show that if a contradiction is true, everything is. If an inference relies on a principle using a noise or shape as a negation sign, to allow contradiction would be to prevent the principle from doing the work the inference would need. If an inference has no negation signs or if it does not use them in the standard way, *p*~p, is not a contradiction in the normal sense; so the inference cannot show that such contradiction in the normal sense makes everything true.

Having looked at an alleged proof of ECQ, let us examine a decision procedure alleged to show that (*p~p)* -> q is valid. ~*p* -> (*p* -> *q*) is truth-functionally valid under normal negation, since ~*p* excludes the only case where *p* -> *q* is false, the case where *p* and ~*q* are both true. So *p* -> *q* evaluates as true as a logical consequence of the truth of premise ~*p*. And if *p* -> *q* and *p* are true, *q* evaluates to true as a logical consequence. But p is also given as a premise. So the standard truth-table definitions of the operators appear to require ECQ.

But if we accept *p*~*p*, ~*p* does not exclude the only case where *p* -> *q* is false, since *p* is still true. So *p* -> *q* is no longer a logical consequence of ~*p*; and the truth of *q* is not established according to the rules of the decision procedure. If *p* -> *q* need not be true, the fact that *q* is a logical consequence of *p* -> *q* and p does not imply that *q* is true. Of course, if we permit contradiction, we can change some other rule or rules of the procedure to make *q* is a logical consequence of *p* -> *q*. But that is my point; the bivalent truth-functional rules will no longer have whatever relation with necessary truths they have now. (On the relation of truth-functions to necessary truth, see Section 17).

Moreover, any method of showing the bivalent truth-functional validity of (*p~p) -> q* under normal negation will make use of other bivalent truth-functional operators. No other operator can do the work that the truth-values of formulas using '->' requires (and vice versa) if we allow contradiction. Should we say that *p* -> *q* is never true, since the truth of ~(*p~q*) does not mean that *p~q* is false and the truth of ~*p* v *q* does not mean that ~(~*p* v *q*) is false; or should we say that *p* -> *q* is always true, since ~*q* does not falsify *q*? There are no grounds for saying either, or equal grounds for saying both, since a unary operator, the negation sign, is not doing the job that the employment of binary operators requires. Allowing contradiction strips the gears of our logical machinery, truth-functional operators in this case, of teeth.

Also, modus ponens (or the law of detachment) is not valid if we permit contradiction; for (*p* -> *q)p* -> *q* is true if and only if (~*p* v *q*)*p* -> *q* is true. But the latter is not true if the truth of *p* does not mean that ~*p* is false. Also, *p* -> *q* is true only if *p~q* is not. But *p* -> *q* and *p~q* can both be true, if ~*q* does not falsify *q*. (And what epistemic goal have we achieved in knowing that *p* -> *q* is true, if we also know that p~*q* is true — or is even able to be true — since *q~q* is true?)

Traditionally, the science of logic was thought to achieve explicit knowledge of the necessary truth of logical principles. Assuming we can have such knowledge, computational procedures establishing that ECQ is valid illustrate the difference between the two epistemic types that I am arguing are needed for knowing computational correctness. Does ECQ express a valid logical consequence? We can be aware that each step in a process establishing ECQ is justified by a rule ordinarily epistemically associated with, or a definition that ordinarily grounds, a logical truth traditionally recognized to be necessary. But permitting contradiction prevents any statement from being a logically necessary truth, to the extent that the statement makes use of negation signs. So, if there is logical knowledge in the sense of knowledge of necessary truths of logic, as I will argue there is, the conclusion that a computational process for showing validity correctly obeys rules or instantiates definitions is not the same as logical knowledge, since it need not be the same as knowledge of any necessary truth of logic.[15]

The fact that we can define the bivalent operators by the matrix method without making negation primitive tells us something important about the epistemic type represented by the matrix computational method itself, but not anything (directly) pertinent to the epistemic type I am calling knowledge of necessary truths of logic. For if we cannot know that what is expressed by 'This attempt to define operators succeeds' excludes its contradictory opposite, or that there is no third choice between excluding or not excluding its contradictory opposite, we achieve no epistemic goal when we think we learn that what 'This attempt to define operators succeeds' expresses, or any other noises express, is true.

Could we have purely *pragmatic* reasons for taking *p* -> *q*, for example, as true under contradiction in some assignments of truth and falsity to *p* and q and false in others? Not if the process of arriving at such a pragmatic decision relies on even one inference thought to be deductively valid; for in making an inference, we rely on knowledge that some argument form has logical validity. And the latter knowledge is not purely pragmatic since, as I argue in the following sections, it is knowledge of the necessary truth of a principle in a logic that does not allow contradiction; for if we allow contradiction, nothing is logically necessary.

A logic with only positive principles, where no consequence depends on negation signs, would not avoid the *epistemic* problem. Again, without a negation sign we cannot claim that a contradiction implies everything. Also, if contradiction is allowed, when we are proceeding as if positive formulas like *p* -> (*p* v *q*) or (*pq*) -> *p* were true, we would do so knowing that they might also not be true. We could decide to use a 'true' positive principle until such time that we found that the principle was also not true. But that decision would not be purely pragmatic, if we were limiting ourselves to positive consequences because of our knowledge that a necessary result of using negation signs while allowing contradiction is that the wheels of our logical machinery would spin but do no work.[16] Nor would it be purely pragmatic if the process of reaching that decision relied on even one inference's being thought to be valid.

As paraconsistent and dialethic logicians are correct to remind us, we have theories, databases, documents and other linguistic structures that contain contradictions and yet do not imply that all sentences are true. (In fact, science makes use of contradictory concepts without implying that all sentences are true; see Section 14.) The reason why is that ECG is false. So in order to prevent the explosion, we do not need a nonstandard logic. To say that is not to say, however, that paraconsistent and dialethic systems cannot be interesting and important for other reasons. (Henceforth, I will not speak of nonstandard 'logics' but of nonstandard 'systems' or 'methods', to confine 'logical' knowledge to knowledge of the necessary. Systems in this sense, of course, can use either a proof-theoretic or semantic method.)

2.

Having illustrated, by way of a classic example commonly considered important, especially by those interested in nonstandard systems, the difference between the two epistemic types, knowing that a computational proof or decision procedure conforms to rules and knowing that the rules or the formulas arrived at by following them express necessary truth, I will now argue that recognizing that a step in a computational process conforms to (is an instance of) a rule requires both kinds of knowledge.

Recognizing the correctness of a computational procedure requires a grasp of what the rules and the formulas justified by the use of the rules, both which may or may not have any connection with necessary truths, are. Again, the rules in a card game like bridge are not necessarily true, but same kind of inference required to know that a computational procedure is a correct application of a rule is required to know that a play in bridge is a correct application of a rule:

If I have a card in the suit that was led, I must play a card in that suit.

I have a card in the suit that was led.

Therefore, I must play a card in that suit.

As this example shows, the point I am making is not arcane; the point is so obvious it is difficult not to overlook it. The rules of bridge necessarily imply that sometimes we must play a card in a particular suit, but the necessity comes from the association of a rule with an inference form whose validity is implicitly known, not from what the rule itself otherwise is. Recognizing the correctness of even just one step in a computational process, requires an inference whose validity we recognize because we know it to be an instance of a necessary truth concerning logical consequence. In this section, I will focus on the fact that knowing that computational processes conform to rules requires a grasp of the validity of an inference and assume that the grasp requires knowledge that a logical truth is necessary. Subsequently, I will show why the grasp of validity requires knowledge of logical necessity.

Since knowledge of computational correctness presupposes a grasp of necessary truth concerning logical consequence, that presupposed knowledge must be acquired other than by computational methods. Without a non-computational grasp of the validity of the inference (and hence of necessity) used to recognize that a computational procedure conforms to a rule, we could not know that the procedure conforms to a rule. This applies as much to procedures in nonstandard systems as it does to standard. The grasp of the validity of an inference required to recognize that procedures conform to the rules of nonstandard — paraconsistent, dialethic, multivalued or whatever — systems relies on a grasp that the opposite of a bivalent truth would be contradictory.

The epistemic inevitability of standard logic does not mean merely that we must start with standard logic, can then use it to build a metalinguistic ladder we can climb to reach a computational method that has nonstandard rules, can kick away the ladder of standard logic after we have constructed the rules of our nonstandard computational method, and can then go merrily on our way no longer relying on standard logic to recognize that computational steps in our nonstandard method correctly conform to our nonstandard rules. Epistemically we can never stop relying on noncomputationally achieved knowledge of truths of standard logic. To recognize that any computational procedure in a nonstandard system conforms to the nonstandard rules, we must continue to make inferences whose validity is always known noncomputationally by knowing necessary truths of standard logic. We cannot kick the ladder of standard logic away. If we ceased using standard logic, we could know that any step in our nonstandard system conforms to the rules.

That noncomputational knowledge of standard logic is epistemically inevitable in this way is a truth that should be of almost no interest to the practicing modern logician; for it is about as close to a trivial truth as one can get without being trivial. But that very closeness-but-no-cigar to triviality makes it important to the philosopher of logic who in addition to recognizing the prodigious success of computational logic, both internally and various of its applications, wants to know the epistemic preconditions for that success.

Even knowledge of the correctness of an operation so fundamental to computational methods as a substitution saving truth or validity requires our grasp of a necessary truth about logical consequence, often modus ponens. Consider this example from Quine:

> Substitution of {*w*: *Gw* v -*Hw*} for '*F*' and '*Gy*' for *p* in
>
> (4) ∀*x*(*Fx* -> *p*) <-> ∃*x Fx* -> *p*
>
> yields:
>
> (5) ∀*x*(*Gx* v -*Hx* . -> *Gy*) <-> ∃*x*(*Gx* v -*Hx*) -> *Gy*
>
> The utility of substitution . . . is as a means of generating valid schemata from valid schemata. E.g., since (3) [not shown here] and (5) were got by substitution in schemata which were seen . . . to be valid, we *conclude* that (3) and (5) are valid. (My emphasis)[17]

'We conclude', that is, knowledge that the newly generated formula is valid according to computationally verifiable rules is a result of an *inference*. As Quine had said earlier about truth-functional schemata 'From the validity of a schema we may *infer*, without separate test, the validity of any schema which is formed from it by (uniform) substitution . . . . of schemata for letters.' (My emphasis)[18]

The rule whose correct application we need to be able to recognize might be something like:

> The result of uniform substitution of schemata for letters in a valid formula is a valid

formula.

The inference to the validity of a new schema would be an instance of modus ponens and could go like this:

(A) (1) If *p q* v ~(*p q*) is a formula gotten from a valid formula by uniform substitution of schemata for letters, *p q* v ~(*p q*) is a valid formula.

(2) *p q* v ~(*p q*) is a formula gotten from a valid formula by uniform substitution of schemata for letters.

(3) Therefore, *p q* v ~(*p q*) is a valid formula.

Or it could go like this:

(1) If '(*p* -> *q*) v ~(*p* -> *q*)' is the result of substituting '*p* -> *q*' for each '*p*' in '*p* v ~*p*', '(*p* -> *q*) v ~(*p* -> *q*)' is a valid formula if '*p* v ~*p*' is a valid formula.

(2) '(*p* -> *q*) v ~(*p* -> *q*)' is the result of substituting '*p* -> *q*' for each instance of '*p*' in '*p* v ~*p*'.

(3) '(*p* -> *q*) v ~(*p* -> *q*)' is a valid formula if '*p* v ~*p*' is a valid formula.

(4) If '*p* v ~*p*' is a valid formula, '(*p* -> *q*) v ~(*p* -> *q*)' is a valid formula.

(5) '*p* v ~*p*' is a valid formula.

(6) Therefore, '(*p* -> *q*) v ~(*p* -> *q*)' is a valid formula.

Epistemically, then, the use of formal methods relies on our ability to recognize that a deductive inference is valid. Rules do not apply themselves. Applying them requires an at least minimal implicit deduction, usually one so obvious it is difficult to notice that it is a deduction. Even when we are doing dialethic logic, knowledge of the legitimacy of the result of a substitution presupposes knowledge of the validity of inferences in standard logic.

N.B. The remainder of this section consists of a hopefully sufficient variety of other examples that those still skeptical will be convinced. If you do not need further convincing that recognizing that a step correctly instantiates a computational rule requires knowledge of the validity of an inference, you can safely skip the rest of this section. Section 3 begins the discussion of why knowledge of inferential validity requires a grasp of logically necessary truth.

Consider this set theoretical example adapted from Stephen Pollard[19]:

(1) (∀*x x*∈*x* -> ∀*y y*=*y*) Assumption

(2) (∀*x x*∈*x* -> ~∀*y y*=*y*) Assumption

(3) ~∀*x x*∈*x* 1, 2 Rule of Negation Introduction

The Rule of Negation Introduction could be:

(NI) If (φ -> ψ) and (φ -> ~ψ) appear on earlier lines, ~φ may be written on a later line.

Recognizing that the formula on line 3 is a computationally correct consequence of the formulas on lines 1 and 2 and the Rule of Negation Introduction, requires an inference that would be an instance of modus ponens such as:

(B) (1) If (∀*x x*∈*x* -> ∀*y y*=*y*) and (∀*x x*∈*x* -> ~∀*y y*=*y*) appear on earlier lines, ~∀*x x*∈*x* may be written on a later line.

(2) (∀*x x*∈*x* -> ∀*y y*=*y*) and (∀*x x*∈*x* -> ~∀*y y*=*y*) appear on earlier lines.

(3) ~∀*x x*∈*x* may be written on a later line.

But even to get from (NI) to (B.1) requires an inference that is an instance of modus ponens at a more fundamental level, an inference such as:

(C) (1) If (∀*x x*∈*x* -> ∀*y y*=*y*) is a substitution instance for φ and (∀*x x*∈*x* -> ~∀*y y*=*y*) is a substitution instance for ψ in (NI) , then if (∀*x x*∈*x* -> ∀*y y*=*y*) and (∀*x x*∈*x* -> ~∀*y y*=*y*) appear on earlier lines, ~∀*x x*∈*x* may be written on a later line.

(2) (∀*x x*∈*x* -> ∀*y y*=*y*) is a substitution instance for φ and (∀*x x*∈*x* -> ~∀*y y*=*y*) is a substitution instance for ψ in (NI)

(3) If (∀*x x*∈*x* -> ∀*y y*=*y*) and (∀*x x*∈*x* -> ~∀*y y*=*y*) appear on earlier lines, ~∀*x x*∈*x*

may be written on a later line.

(D) [Proofreader: I just discovered that the example I had called (D) was redundant with (B). Since this was a last minute discovery, I did not want to change the letters used to name subsequent examples at this time, for fear of invalidating any cross-references to those examples. So please ignore any cross-references to example (D). But please flag any other invalid cross-references that you find.]

The fact that the minor premises of arguments (A), (B), (C) and (D) are empirically verifiable does not dispense with the need to grasp the validity of an inference to know that a rule concerning computational procedures, such as making a substitution or introducing a formula beginning with a particular sign, here, ~, has been applied correctly. What these examples illustrate is that recognizing that a rule has been applied correctly requires, in addition to a grasp of what the rule and any formulas to which it is applied are, a recognition of the validity of an inference form such as modus ponens.

That recognizing computational correctness depends on grasping of the validity of inferences may be even clearer in the case applying semantic definitions of logical constants. The following examples, adapted from Matthew McKeon's adaptation of Tarski,[20] illustrate just some of the kinds of inferences that can be required for applying semantic definitions in establishing a computationally verifiable idea of logical consequence. I will use ordinary English where appropriate for abbreviating a lengthy series of inferences and for making the pre-computational character of the inferences more perspicuous. As a result I will use 'if . . . then' constructions as well as '. . . -> . . .' formulas. I will assume that the interpretation of formal sentences uses a non-empty domain, the empty variable assignment and a variable assignment that is an extension of it.

To be shown is that a sentence is a model-theoretic consequence of the empty set of sentences if and only if any arbitrary structure for its language is a model of the sentence. Recognition of that truth requires recognition of the validity of inferences like:

(E) 1) If any arbitrary structure (for the semantically defined computational language) is a model of $\forall x M(x) \to \exists x M(x)$, $\forall x M(x) \to \exists x M(x)$ is a model theoretic consequence of the empty set of premises.

2) Any arbitrary structure (for the semantically defined computational language) is a model of $\forall x M(x) \to \exists x M(x)$.

3) $\forall x M(x) \to \exists x M(x)$ is a model theoretic consequence of the empty set of premises.

And

(F) 1) If sentence X is a consequence of the empty set of sentences, every structure for X's language is a model of X.

2) X is a model-theoretic consequence of the empty set of sentences.

3) Every structure for X's language is a model of X.

The computationally verified process of arriving at premises for inferences like (E) would include showing that any arbitrary structure U is a model of $\forall x M(x) \to \exists x M(x)$. Recognition of the success of such a verification would include recognition of the validity of inferences like:

(G) 1) If U is a structure whose domain is D, D is non-empty. [By the definition of 'structure'.]

2) U is a structure whose domain is D.

3) D is non-empty.

Or

(H) 1) Every structure has a non-empty domain.

2) U is a structure.

3) U has a non-empty domain.

And

(I) 1) Every non-empty domain has at least one element d. [By the rule defining 'non-empty domain'].

2) D is a non-empty domain.

3) D has at least one element d.

Are these examples almost trivial? Precisely. That shows why it is so easy to miss the dependence of computational methods on the pre-computational grasp of logical validity. And further (where v is any unspecified variable):

(J) 1) If (U is a model of ∀*x*M(*v*)) is a result of substituting M(*x*) for *v*? in the rule for ∀, then U is a model of ∀*x*M(*v*) -> U satisfies M(*v*) for every element d in D.

2) (U is a model of ∀*x*M(*v*)) is a result of substituting M(*v*) for *v*? in the satisfaction/truth rule for ∀.

3) U is a model of ∀*x*M(*v*) -> U satisfies M(*v*) for every element d in D.

And (where *x* is an assigned variable)

(K) 1) If ((U is a model of ∀*x*M(*x*)) -> U satisfies M(*x*) for every element *x* in D), then U is a model of ∀*x*M(*x*) -> U satisfies M(*x*) for at least one element *x* in D. [By the rule that a structure has a non-empty domain and substitution in a version of dictum de omni pre-computationally known to be necessary.]

2) U is a model of ∀*x*M(*x*) -> U satisfies M(*x*) for every element *x* in D. [J.3. But even recognizing the computational legitimacy of repeating a sentence already established at one point as a premise at another point involves an inference using a computational rule to that effect as a premise.]

3) If U is a model of ∀*x*M(*x*), then U satisfies M(*x*) for at least one element *x* in D.

And

(L) 1) If (U satisfies M(*v*)) is a result of substituting M(*v*) for '*v*?' in the satisfaction/truth rule for ∃, then U satisfies M(*v*) -> U is a model of ∃*x*M(v).

2) (U satisfies M(*v*)) is a result of substituting M(*v*) for '*v*?' in the satisfaction/truth rule for ∃.

3) U satisfies M(*v*) -> U is a model of ∃*x*M(*v*).

And

(M) 1) If ((U is a model of ∀*x*M(*x*) -> U satisfies M(*x*)) and (U satisfies M(*x*) -> U is a model of ∃*x*M(*x*))), then U is a model of ∀*x*M(*x*) -> U is a model of ∃*x*M(*x*). [Abbreviating a series of inferences from the rules of truth for sentential connectives and substitution.]

2) ((U is a model of ∀*x*M(*x*) -> U satisfies M(*x*)) and (U satisfies M(*x*) -> U is a model of ∃*x*M(*x*))). [J.3 and L.3 and substitution in the rule of truth for sentential connective ‘and’.]

3) U is a model of ∀*x*M(*x*) -> U is a model of ∃*x*M(*x*).

And

(N) 1) If U is a model of ∀*x*M(*x*) -> U is a model of ∃*x*M(*x*), then either ~(U is a model of ∀*x*M(*x*)) or U is a model of ∃*x*M(*x*). [By the rule of truth for -> and substitution.]

2) U is a model of ∀*x*M(*x*) -> U is a model of ∃*x*M(*x*). [M.3]

3) Either ~(U is a model of ∀*x*M(*x*)) or U is a model of ∃*x*M(*x*).

And

(O) 1) If either ~(U is a model of ∀*x*M(*x*)) or U is a model of ∃*x*M(*x*), then U is a model of ∀*x*M(*x*) -> ∃*x*M(*x*). [By the satisfaction/truth rule for -> and substitution.]

2) Either ~(U is a model of ∀*x*M(*x*)) or U is a model of ∃*x*M(*x*). [N.3]

3) U is a model of ∀*x*M(*x*) -> ∃*x*M(*x*).

And

(P) 1) Any sentence of which any arbitrary structure U is a model is a sentence of which all structures are models. [Here a version of dictum de omni is a premise in an inference relying on the pre-computational grasp of the validity of dictum de omni as a rule of inference.]

2) ∀*x*M(*x*) -> ∃*x*M(*x*) is a sentence of which any arbitrary structure U is a model. [By *inference* from O.3.]

3) ∀*x*M(*x*) -> ∃*x*M(*x*) is a sentence of which all structures are models.

And

(Q) 1) Any sentence of which all structures are models is a model-theoretic consequence of the empty set of premises. [And is this premise an epistemically ‘trivial’ truth, or does our epistemic grasp of it's truth require further inferences from different 'trivial' truths?]

2) $\forall x$M(x) -> $\exists x$M(x) is a sentence of which all structures are models. [P.3]

3) $\forall x$M(x) -> $\exists x$M(x) is a model-theoretic consequence of the empty set of premises.

Notice that despite the absence of any modally defined terminology in the computational procedure thus verified, our pre-computational grasp of the soundness (the logical validity and the truth of the premises due to the rules of the system) of the argument causes the epistemic result that we know that NECESSARILY any arbitrary structure MUST be a model of $\forall x$M(x) -> $\exists x$M(x). And it causes us to know, not just that all structures happen to be models of $\forall x$M(x) -> $\exists x$M(x), but that all *possible* structures *must* be models of it. And if we did not pre-computationally know that, we would not consider the result of the computational procedure an acceptable computational account of logical consequence.

I have used dictum de omni in examples D, E, G, L and M to show that conditionals like modus ponens are not the only necessary logical truths the grasp of which we may rely on for recognizing computational correctness.

3.

How do we recognize that a particular argument is valid, that is, how do we recognize the relation between two sentences that allows us to know that if these two are true, another sentence must be true? Answering that question will show why the grasp of validity requires knowledge of necessary truth.

Consider the following:

(1) If *p*, then *q*.

(2) *p*

Presumably, we are aware of an inferential relation between (1) and (2) that lets us know that if they are true, the following is true:

(3) *q*

Knowledge of this inferential relation cannot consist in our knowing an inferential relation between sentences like the following:

(1') If 'If *p*, then *q*' is true and *p* is true, then *q* is true.

(2') 'If *p*, then *q*' is true, and *p* is true.

Knowing the inferential relation between two sentences that gives us knowledge of a conclusion cannot consist in knowing another inferential relation between (i) a fourth explicitly formulated sentence, the law stating that the conjoined truth of the first two sentences requires the truth of the conclusion, and (ii) the conjoined truth of the first two sentences. If knowing an inferential relation requires knowing another inferential relation, knowing any inferential relation requires knowing an infinite series of such relations.

The preceding is, of course, a version of Lewis Carroll's 'Achilles-Tortoise' paradox. It is sometimes said that Carroll's paradox results from confusing rules of inference with premises. But more must be said. Quine, for example, would probably want to add that we cannot avoid the regress by distinguishing between the premises of an argument and its inference rules, since we can no more make a hard and fast distinction between rules and premises than we can, according to him, between analytic and synthetic statements. (Section 10 will reply to Quine about such distinctions.) But more fundamentally, the distinction between rules and premises does not itself tell us how our knowledge of what is expressed by rules is related to our knowledge of what is expressed by the premises when we know the validity of an inference.

For one thing, the grasp of a rule of inference that recognizing the validity of an inference requires must be implicit. By 'implicit' I mean that we grasp the truth of, for example, modus ponens without formulating it in statements distinct from the concrete instance in which we use it. We do not grasp the truth of a statement like 'If statement 1 implies statement 2, and statement 1 is true, then statement 2 is true', apart from and as opposed to grasping the validity of an inference like 'If the road is wet, it's slippery; and the road is wet; therefore, it is slippery'. Carroll's paradox shows that an infinite regress occurs if knowledge of the success of an inference requires anything more than an understanding of what its premises are and what its conclusion is. So a knowledge of modus ponens as expressed in a sentence distinct from the sentences making up such an inference would have to be irrelevant to our grasp of the inference's validity.[21]

For another thing, the infinite regress occurs unless the validity of an inference is knowable simply by knowing what its premises and conclusions are. If we can't recognize validity that way, we need another inference whose validity is only recognized by another inference, and so on. But a grasp of validity does not require a grasp of the truth-values of the premises; so the simultaneous knowledge of 'what the premises and conclusion are' need not include knowledge that they are truths. The recognition of what the premises are that causes recognition of validity must be merely the recognition of what is expressed by the words, or at least some of the words, of the premises; so the recognition must require only a grasp of how at least some of the words making up the argument are being used. For the success of more complex inferences to be knowable, the success of less complex inferences making them up must be knowable simply from an understanding of how some of the words in the premises and conclusions of the less complex inferences are being used.

Part II will discuss our *knowledge* of the truth of inference principles further. But the consequences of Carroll's paradox provide enough material to explain why knowledge of inferential validity must be knowledge of necessary *truth*. (That logical knowledge is of the necessary does not mean that the truth preserved by valid inferences must be modal. Of course, since the opposites of all truths are either contradictory or non-contradictory, all truths are either necessary or contingent as defined by the contradictoriness of the opposite, and so all truths are modally characterized in one way or another whether or not they use modal terms and whether or not we know their modality, implicitly or explicitly, when we know their truth.)

I have called knowledge that a formula results from a correct application of a computational rule computational knowledge, as opposed to logical knowledge of a necessary truth that a rule may be epistemically associated with. In addition to knowledge of the logically necessary, computational knowledge presupposes knowledge of two kinds of contingent truths: (1) that a rule is what it is and (2) that a state of affairs that the rule concerns occurs [for example, that a formula or schema of a certain form appears on a line of a proof — as in (B) of Section 2, or that a formula is a substitution instance of a schema or of a schematic letter used in a rule to represent the formulas which the rule concerns — as in (C), or that I have a card in the suit that was led — as in the bridge example]. If we are using a conditional to express the rule, (2) would be the contingent knowledge that the antecedent or consequent, as the case may be, of the conditional is satisfied.

Since rules do not apply themselves, the other kind of knowledge presupposed by the recognition of computational correctness is knowledge of the validity of an inference concluding that a specific step is correct according to a rule. That knowledge of inferential validity includes implicit knowledge of a necessary truth of logic, like modus ponens, stating the validity of the inference and explicit knowledge a rule and the facts to which the inference applies the rule. The goal of traditional and classical modern logic was *justified explicit knowledge of the necessary truth of inferences principles that are only implicitly known to begin with*.

Epistemically. the difference between knowing that three statements like (i) 'If it is raining, the road is slippery', (ii) 'It is raining', and (iii) 'The road is slippery' happen to be simultaneously true, on the one hand, and knowing that (iii) is a logical consequence of (i) and (ii), on the other, is that even when we don't know that (i) and (ii) are true, we know that on the hypothesis that (i) and (ii) are true, (iii) is true. The latter knowledge, logical knowledge, is knowledge of the necessary. That it is knowledge of the necessary follows from the fact that it is caused simply by knowing that for which 'if . . . then' happens to be used, namely, a certain kind of relation, which I will call 'C' between the possible truth-values of statements. If we are able *intend* to use 'if . . . then' even to just hypothesize (i), the statement that C holds between the possible truth-values of (ii) and (iii), we are capable of recognizing that the further hypotheses that (ii) is true and (iii) is false *contradict* what we hypothesized relying only on that for which we intend to use 'if . . . then', relation C.

I am calling that for which we use 'if . . . , then' C to avoid using words like 'implication' or 'entailment' in this immediate context. That for which we pre-computationally use 'if . . . then' might sometimes, even normally, include relations like relevance or causality. But we do not need to know everything that may or may not be included in that for which we use 'if . . . then' to know that our grasp of the validity of 'if . . . then' inferences is indeed a grasp of the necessary truth of modus ponens. Whatever may be included in that for which we use 'if . . . then', our certitude of the validity of those inferences is caused only by our grasp of that for which we use 'if . . . then', relation C. For whatever relation C is, that it holds is all we intend to communicate when we assert 'If it is raining, then the road is slippery', *as opposed to* intending to communicate the truth of 'It is raining' and/or 'The road is slippery'. And if we have a sufficient grasp of what C is to intend to hypothesize the truth of (i), we have a sufficient grasp of C to recognize that the truth of (ii) and falsehood of (iii) would contradict that hypothesis.

Or, and this is what matters here, that *can* be all we intend to communicate. For the sake of our 'purposes at hand', we can make the assumption that C is all we intend to communicate by 'if . . . then' at those times when we can, for the sake of pursuing the kind of knowledge we happen to call 'logical', abstract and explicitly formulate modus ponens from cases like (i), (ii) and (iii). For we are then articulating a rule whose truth we can *intend* not to depend on the truth of (i) and (ii). And we can abstract from the actual truth-values of (i) and (ii) without abstracting from their possible truth-values, that is, while still considering them as bearers of possible truth-values. And when we abstract modus ponens from cases like (i), (ii) and (iii), if we were not intending (i) to ignore the actual truth-values of (i) and (ii), what we were using the words or symbols of modus ponens for at that time would be irrelevant to our discussion, since what neither traditional logic nor modern logic *chooses*, for their purpose at hand, to study when studying what they call 'valid inference' happens to be something that depends on the actual truth-values of a conditional's antecedent and consequent. But they are choosing to study something that requires them to hypothesize that antecedents and consequents are bearers of truth-values, and so requires them to consider those statements as possible bearers of truth-values.

Nor is that choice arbitrary. We choose to study such actual-truth-value independent relations because we have first found that they do in fact occur! Why they occur and how we know that do is the subject of Part II. Here I am only concerned show that we do in fact find truths — which I will argue happen to be about such relations — whose opposites we know to be contradictory, and so to satisfy the definition of 'necessary truth' that most of past philosophy happens to use. (For what Part II will say about logical knowledge, it will not matter whether that for which we normally use 'if . . . then' includes more than a truth-functional relation between the possible truth-values of the antecedent and consequent, as long as that for which we can intend to use it at least includes a strictly for a truth-functional relation. If we are able to intend its to use it for something that at least includes a truth-functional relation, we are able to recognize that the relation's not holding between the truth-values of (ii) and (iii) would contradict what we intend to hypothesize by using 'if . . . then' in (iii). And if we were not able to so intend the use of 'if . . . then', conditionals would be irrelevant to the chosen purpose that we happen to call 'logical knowledge'.)

Why, then, does the recognition that an 'if . . . then' inference is valid require the implicit grasp of the necessary truth of modus ponens? Our belief in the truth of modus ponens is caused only by a recognition of what C is (not in any way a perfect recognition, but a recognition sufficient for us to make whatever C is that for which 'if . . . then' constructions are used); for our grasp of the fact that the truth of (ii) and falsehood of (iii) contradicts what we hypothesize by (i) derives solely from our understanding of what we are hypothesizing in (i), C. So the evidence for the truth of modus ponens consists only of what C happens to be. Since the evidence consists only of what C happens to be, knowledge of the evidence consists of knowing that, if modus ponens is not true, what C is is not what C is; C is not C. In other words if modus ponens is not true, C, that for which we happen to be using 'if . . .

then', both is C, since modus ponens includes C and C causes modus ponens to be true, and is not C, since we are also assuming that while modus ponens is what it is, and so includes C, modus ponens is not true.

What C is causes

(1) If (ii) then (iii), and (ii), so (iii),

which hypothesizes C, 'If (ii) then (iii)', to be true. But since C is what it is, hypothesizing that (ii) is true and (iii) is false contradicts (1) and so amounts to assuming that modus ponens is not true. So we are simultaneously hypothesizing that (1), modus ponens, is true and is not true. And since we understand that the cause of modus ponen's truth, what C happens to be, is what it is, we can recognize that the simultaneously hypothesizing the truth of modus ponens together with the truth (ii) and the falsehood of (iii) amounts to simultaneously hypothesizing that what C is is not what C is. That is why knowledge of the truth of modus ponens is knowledge of the necessary: it is knowledge that its denial is contradictory. If the evidence for modus ponens consisted of more than the fact that, if modus ponens is not true, what C is is not what C is, the evidence does not consist solely of the fact that C is what it is. And if the evidence does not consist solely of C's being what it is, relation C's holding between the truths of (i) and (ii) and the truth of (iii) is not what we are intending to communicate or just hypothesize, when we communicate or hypothesize the truth of (i), as opposed to the actual truth of (ii) and (iii).

So what C is makes modus ponens true in such a way that (a) knowing what C is and (b) knowing that what C is is that for which we use 'if . . . then' is sufficient for us to know that modus ponens is true. For in recognizing the truth of modus ponens we are recognizing that if it is not true, the evidence for its truth both is and is not what it is: what C is, which happens to be the evidence for the truth of modus ponens, is not what C is at the same time that it is what C is. Hence the falsehood of modus ponens would require a contradiction to be true, and modus ponens is necessarily true.

'If modus ponens is not true, C is not C' does not mean that something that was previously C would have changed and become something that is now not C, while retaining the rest of the identity it has other than that of being C. That assumption would not make anything simultaneously C and not C. Rather, the assumptions are that what we call 'modus ponens' includes C and that *modus ponens remains what it is throughout the discussion*. Therefore, if what C is is the evidence for the truth of modus ponens, modus ponens, because it includes C, can fail to be true only if what C is is simultaneously not what C is; if modus ponens can fail to be true for any other reason, our knowledge of what C is is not the sole cause of our certitude that modus ponens is true. So if we know that modus ponens is true, we know it only by knowing that the opposite requires what C is to also not be what C is; that is, we know modus ponens is true only by knowing that its opposite is contradictory. And if we do not implicitly know, just by knowing that for which 'if . . . then' is used, that modus ponens is true, we don't know that the kind of inference made explicit by modus ponens is valid. So we can recognize that an inference is valid if and only if we can recognize, implicitly at first and then explicitly, that a statement that it is valid is true on penalty of contradiction: necessarily true.

Notice that, as in the case of negation and disjunctive syllogism, the necessity of modus ponens does not depend on the relation between 'if . . ., then' and C being what it happens to be; it depends on C being what it happens to be. It does not depend on the lexicological relation between 'if . . ., then' and C being what it is; it depends on the logical relation hypothesized between antecedent and consequent, the relation C, being what it is. Quine might find a 'fallacy of subtraction' in making the distinction between knowledge of what C is and knowledge of what the relation between 'if . . ., then' and C is. Finding this fallacy might be occasioned by the worry, typical of Quine, that the distinction would could require us to open a Pandora's box of all sorts of illegitimate mental states and acts. In particular, it might lead us back into a psychology of thought functioning independently of language. We need not worry. When we hear and understand 'Your mother has died', our shock and grief is not caused by our conscious relation to what the noise 'died' is; it is caused by the fact that we have a conscious relation to what death is. But we need not conclude that this way of being related to what death is is causally independent of our ability to use language.

4.

Without attempting a complete definition of the science of logic, we can say that both traditional logic (TL) and classical modern logic (CML) aimed to achieve explicit knowledge of the kind of necessary truths of which we have implicit knowledge whenever we knowingly make a valid inference. But if logic is the science that seeks explicit knowledge of the necessary truth of principles of valid inference, recognition that a computational step conforms to rules of computation is not enough to give us logical knowledge. Merely recognizing, for example, that a formula expressing modus ponens results from using rules correctly does not tell that the formula expresses a necessary truth. We can design rules, like the rules of bridge, by the use of which we can infer a result that need have no connection with any necessary truth other than the inference principle whose implicit knowledge caused our grasp of the validity of the inference. Since computationally verifiable rules need not express necessary truths, recognition of computational correctness is useful for achieving that common goal of TL and CML only if we can also non-computationally recognize a connection between some rule or rules specifying correctness in computations and some necessary truth or truths, such that we can also non-computationally recognize that if a formula resulting from using the rules correctly does not express necessary truth, the rules do not have their known connection with necessary truth. The genius of CML has been its ability to formulate rules serving that purpose.

Again, a computational method can use directive rules, like ‘Delete a true formula from a conjunction’, that are neither true nor false. But if we are using the method to show logical validity, we choose at least some of the rules because we know the necessary truth of principles like ‘Deleting a truth from a conjunction preserves truth-value’, on the basis of knowing a way that we precomputationally use ‘and’.

An objection might be that while it is appropriate to speak of connections between a rule like modus ponens, or an introduction or elimination rule in a natural deduction system, and a logically necessary truth, it is not appropriate to speak in the same sense of connections between definitions and necessary truths. Definitions, not being statements, are neither true nor false and so neither necessarily true or false or contingently true or false.

But we can so choose definitions that we can know (for reasons to be discussed in Part II) that a computational method verifies a formula like, for example, '∀*x*M(*x*) -> ∃*x*M(*x*)' only if that formula expresses a necessary truth; for if it does not express a truth, the following satisfaction conditions used in defining'∃', '∀', and '->', respectively, are not what they, at the same time, are:

(1) A structure satisfies ∃*x*M iff at least one element of the domain satisfies ∃*x*M.

(2) A structure satisfies ∀*x*M iff all the elements of the domain satisfy ∀*x*M

(3) An structure satisfies A -> B iff either it does not satisfy A or does satisfy B.

Such definitions do not express necessary truths but contingent stipulations for the use of signs, though stipulations that are intended to remain constant throughout the use of a particular computational method. But once we have stipulated those definitions, relations between the satisfaction conditions of two or more formulas using those constants either hold necessarily or do not. And for standard systems, we so choose the conditions for defining the constants we choose to call 'logical' that we can recognize computationally that at least some formulas using them express truths made necessary by relations between those conditions. That is, we so define the symbols that if certain formulas employing them are not true, the conditions defining them both are and are not what they are. They are what they are, since in stipulating them we assume, at least implicitly, that the stipulations do not change, and are not what they are, since we are assuming for the sake of argument that formulas they cause to be true by being what they are are not true.

We can define some logical constants, such as '&' and 'v', so that we know the use of each is identical with (at least) a one of the ways we precomputationally use certain words, such as ‘and’ and ‘or’, respectively. Given what we know about that use of ‘&’ and ’and’, for example, we can then recognize that while ‘A & B’ is true, ‘A’ cannot not be true, on penalty of contradicting what we assume in assuming that ‘A & B’ is true. And since the other terms in those definitions are uninterpreted symbols, we can know that, under penalty of contradiction, certain wffs using ‘&’ and ‘v’ according to the definitions are true on any interpretation of the other symbols in these wffs; for we can know that these wffs must be true solely because that for which we are using the symbols ‘&’ and ‘v’ are what they respectively are.

But definitions stipulated for the sake of acquiring knowledge of computational correctness need not make the use of a logical constant identical with a way we use any word precomputationally. If so, some wffs that are necessary consequents of those definitions need not be among the necessary truths we must know pre-computationally in order to recognize valid inference and so recognize computational correctness. If -A -> (A -> B) is not a correct result of applying the standard definitions for ‘-‘ and ‘->’, the truth conditions for which ‘-‘ and ‘->’ are used are and what they are and are not what they are. And if that for which we precomputationally use ‘if . . . then’ is the same set of truth conditions, the falsehood of ‘A’ validly implies the truth of ‘If A, then B’. But the necessary computational correctness of -A -> (A -> B) establishes nothing about whether “If both ‘A’ and ‘B’ are false, then ‘If A then B’ is true” is an inference valid on penalty of that for which we precomputationally use ‘if . . . then’ not being what it is while being what it is.

In nonstandard systems, some of the rules or definitions are not chosen because of their connection to precomputationally known necessary truths. But all rules are chosen so that we can recognize that if a computational result is not what it is, some use that has been stipulated for a sign occurring in the result is not what it is. Stipulated definitions are not necessary truths but can be chosen so that we know, by means of an inference relying on implicit awareness of some necessary truth, that if a result is not what it is, the defining condition (not the relation of the definition to that condition, but the condition) is not what it is at the same time that it is what it is. In this way, we construct rules and definitions such that some wffs cannot fail to preserve some designated value in multivalued systems, or to preserve truth on some intensional possible world semantics.

In order to have explicit knowledge that a computational result expresses a necessary truth, we must begin from knowledge of the connection with necessary truth of some computational rule or set of rules. But only when knowledge results from the use of such rules do computational methods give us the kind of knowledge that is the goal in both TL and CML: explicit knowledge of necessary logical truth about the validity of inferences. The presupposed knowledge of the connection between rules and necessary truth is not the knowledge I am calling computational, that is, knowledge that something is the result of a correct application of the rules. For in the final analysis the source of our knowledge of the connection between a rule and necessary truth cannot be the merely computational grasp that a process correctly instantiates rules. The ability of a computational process to result in knowledge of the necessity of a truth presupposes that we already have a grasp of the connection between a rule or rules of the computational method and necessary truth. Computational methods can give us knowledge of necessary truth only in dependence on some other way of knowing necessary truth. So, computational methods are useful in logic only because there is such a thing as noncomputational knowledge of necessary truth. Without noncomputational knowledge of necessary truth we could not recognize that the result of a computationally correct procedure expresses a necessary truth.

Not all the necessary truths whose knowledge is presupposed by our use of computational methods must be truths about forms of inference. A matrix definition of an operator is a useful tool for logical verification only because we know that the matrix's assignment of symbols for a finite number of values, like 'T' and 'F' or '0' and '1', to symbols for a finite number of formulas, like *p* and *q*, *exhausts all the possibilities*, that is, because we know that additional assignments different from, rather than just repeating, those shown in the matrix are im-possible. If the rules permit only two values, say, '0' and '1', and permit only one value to be assigned to any formula at one time, we can know that there can be four (2^2) and only four different combinations of values for any two formulas: *p*/0, *q*/0; *p*/1, *q*/0; *p*/1, q/1; p/0, q/1. And so we can also know that corresponding to each of the latter combinations there can be 16 (4^2) and only 16 different combinations of four 0/1 values in the matrix: (1, 0, 0, 0; 1, 1, 0, 0; 1, 1, 1, 0; etc.).

If we did not have that kind of knowledge, matrices would be useless for logical verification. For if we did not know that the sixteen possible combinations of four 0/1 assignments exhaust all the possible combinations of different 0/1 assignments to two formulas, we could not use matrices to learn, by a finite number of steps, that a formula using operators defined by this method was assigned 1 by all possible combinations or 0 by all possible combinations. And we would not consider such formulas computationally verified if we could not recognize that they are true on all possible assignment of truth-values to their atomic formulas. Recognizing that amounts to knowing that if a matrix did not assign all possible values to the component sentences of a sentence using an operator, either the matrix or the truth-conditions defining the operator both are and are not what they are. Again, these remarks should be all but trivial for those whose interest is doing logic by computational methods. But that all-but-triviality is precisely why those whose interest is the epistemic presuppositions of doing logic by computational methods should find them important. (For further discussion of truth-functional definitions and necessity, see Section 17.)

In these cases, we need to know necessary truths of mathematics in order to verify necessary truths of logic by computational methods. Using mathematical induction in computational proofs also allows us to grasp in a finite number of steps that all the possibilities for integers are covered; for knowing the definitions of 'n', '+', and '1' can cause knowledge of the necessary truth that n + 1 can be any of the infinite possible integers greater than 1. But with respect to such mathematically necessary truths used to know that a computational method for logic covers all the possibilities, is it still true that the knowledge presupposed to the computational knowledge is itself gained *noncomputationally*? Don't we grasp mathematically necessary truths by computation? For example, our ways of verifying the results of the arithmetic operations are decision procedures, and proofs in traditional geometry employ axioms sets in verifiable steps.

Yes, but part of my characterization of 'computational' methods for the purposes of this inquiry was that they are rule-following methods where the rules need not be epistemically associated with necessary truths. If mathematical knowledge is, ultimately, derived from an explicit grasp of the necessary truth of statements, as I would argue it is, that knowledge is not computational in my narrower sense. The reason for stipulating that logic's computational rules, unlike, say, the computational rules of arithmetic, need not be epistemically associated with necessary truths is that my arguments are, among other things, directed against anyone who thinks that modern logic in general, or some particular development(s) in it, eliminates dependence on knowledge of the necessary from logic. But going into the exclusion of mathematics from the meaning of 'computational', and the reason for it, at the beginning would only have needlessly complicated an already too lengthy *met en scene*.

On the other hand, in addition to her explicit grasp of mathematically necessary truths, or of the connection with necessary truths of the rules for arranging strings of marks in a mathematical decision procedure, a mathematician can only grasp the validity of inferences using those rules if she also has an implicit grasp of the necessary truth of principles of which her inferences are instances. To grasp the correctness of any step in a computational procedure for establishing a mathematical truth, for example, that there can only be 2^2 combinations of assignments of two values to two formulas, she must have an implicit awareness of the necessary truth of whatever logical truths about valid inference the procedure would instantiate, just as she must for computational procedures in logic. Otherwise, an infinite regress would arise for her just as would for the logician. So our use of mathematical reasoning to verify that a set of rules for logical computation covers all the possibilities can never eliminate the need for the noncomputational recognition of the necessary truth of inference principles. Our grasp of the correctness of both logical and mathematical computational steps presupposes a noncomputational grasp of logically necessary truth, a grasp not causally dependent on the empirical verification that a step or series of steps is correct according to the rules of a method.

Of course, another way to obtain the knowledge the all the possibilities are covered is by adding a 'And nothing that is not described by one of the preceding conditions is a . . .' stipulation to a list of rules, for example, to a recursive definition. But we use such stipulations because (1) we *know* that they make it true that all the *possibilities* are covered because we know that it would *contradict* the hypothesis if there were other instances, and (2) we know beforehand that ensuring that all the possibilities are covered is a *necessary* epistemic causal condition for the method is to be successful, at least successful in determining logical validity.

And of course, computational methods also require rules, such as syntactical formation rules, which need have no more connection with necessary truth than do the rules of bridge. But applying even syntactical rules requires an implicit grasp of necessary inferential validity, just as applying the rules of bridge do. Even an operation as simple and basic as substitution provides a sufficient example, as we saw in Section 2, and to use definitions like semantic definitions (1), (2) and (3) above we need to know that letters like 'A' and 'B' are acting as placeholders for which wffs are to be substituted. So even rules with no independent connection to necessary truth are chosen to be useful for knowing that when a wff like '∀*x*M(*x*) -> ∃*x*M(*x*)' is verified by the use of the method, it expresses something made true solely by the fact that the definitions of its logical constants (which definitions presuppose the rules being what they are) are what they are.

5.

One important upshot of the distinction between knowledge of logical necessity and computational knowledge is that the following relation holds between logic in the traditional sense (TL) and classical modern logic (CML): Computational methods are indispensable *tools* for determining the validity of forms of inference, that is, for achieving TL's goal of explicitly knowing the validity of forms of inference. These tools are indispensable because they are incomparably clear, precise, comprehensive and progressive. By 'clear', I mean the rules and definitions are neither vague nor ambiguous. By 'precise', I mean rigorous; CML reduces the verification of validity to the recognition that each step in a process satisfies one or more of a finite set of rules mechanically specifying the formation, positing, combining and detachment of finite sets of marks. By 'comprehensive', I mean that CML's methods can verify the validity of some inference forms that we pre-computationally know to be valid but that the methods of TL could not show to be valid (apparently[22]). By 'progressive', I mean that the ways in which CML can expand our knowledge of the validity of inference forms may be limited only by our ingenuity in constructing computational methods.[23]

The preceding paragraph is meant to express facts about the relation between TL and CML that everyone already knows. Not so the following statements which, I am arguing, are also true. To the extent that the recognition of neither the correctness of computational procedures nor of what the rules are that make them correct need amount by itself to the recognition of any necessary logical truth, the tool that CML provides for achieving the goal of TL is a different epistemic type from TL. Here, I am not just repeating the commonplace that, relative to other kinds of knowledge, logic is knowledge of a tool that those other kinds of knowledge use: inferential validity. That is true; logical knowledge is knowledge of that tool. But relative to logical knowledge, therefore, computational knowledge is a tool for knowledge of a tool, a tool once removed. In class, Putnam called Quine's requirement that, for referring, sentences be regimented in Fregean fashion 'the sacrilization of logic'. It does not lessen the importance of Putnam's insight to note that, strictly speaking, Quine was not sacrilizing logic but logic's use of a particular tool.

Epistemically speaking, the role of computational methods for achieving knowledge of necessary truths of logic is analogous to the role of mathematics for physics. The difference between knowledge of computational correctness and knowledge of necessary truth does not prevent computational knowledge from being an indispensable tool for achieving knowledge of necessary truth any more than the difference between mathematical knowledge and physical knowledge prevents mathematics from being an indispensable tool for physics. Knowledge of mathematical truths is not the same as knowledge of truths of physics, but we can hardly get anywhere in physics without mathematics. Likewise, knowledge that steps in a computational process satisfy rules is not the same as knowledge that any rule or formula reached by means of the steps expresses something logically necessary, but computational methods of inference are just as much indispensable tools in logic as mathematics is in physics. Logic can start without them but can hardly get anywhere without them in comparison to where it can get only with them. CML is a way of achieving the goal of TL, but CML does it by means of an incomparably powerful tool, a tool without which you cannot get any further in logic than you can get in physics without mathematics.

To describe mathematics as a tool that serves something other than itself is not to imply that mathematical knowledge is not intrinsically valuable. Not only is mathematics an entirely legitimate kind of knowledge on its own, but if it was not entirely legitimate on its own, it would not be a useful tool for physics. Relative to the knowledge physics acquires, mathematical truths are 'only' a tool (an indispensable and incomparably powerful tool), a means, not the end; mathematical truths are not the kind of truths that physicists seek to discover and verify. But for mathematicians mathematical truths are not only a tool; they are the truths that mathematicians seek to discover and verify.

Just as mathematics is entirely legitimate subject on its own, not merely as an aid to physics, so the study of computational methods is an entirely legitimate subject on its own, not merely as an aid for verifying the validity of inferences in standard logic. And just as the mathematical truths physics uses are truths that mathematics studies by mathematical methods, so CML not only uses computational methods to verify logical validity, but it studies computational methods by computational methods themselves. In fact logicians have for some time been rightfully more interested in the study of computational methods by computational methods than in simply using computational methods to verify the logical validity of formulas. Similarly mathematicians are more interested in studying mathematical truths for their own sake than as aids to other things. So to describe knowledge of computational correctness as a tool that serves something other than itself, knowledge of necessary truths about forms of inference, is not to imply that CML is just of a servant of TL.

Among the topics mathematicians study for their own sake are things that have no foreseeable applications, scientific or practical. But it is unlikely that mathematics would have advanced as far as it has had it been proven so useful in fields other than mathematics. Likewise, it is unlikely that modern logic's interest in non-standard systems and in using computational methods to study computational methods themselves would have advanced as far as it has if computational methods had not first proven so powerful in verifying the validity of inferences in consistent and bivalent logic. Why would anyone have tried to prove the consistency and completeness of computational systems, for example, if those systems were not already known to be such a powerful method of establishing inferential validity?

But there is this difference between mathematics and physics, on the one hand, and computational methods and logical knowledge, on the other: With respect to knowing that truths are necessary, the roles of the tool and the field using the tool are reversed. Mathematics, the tool, knows truths to be necessary while the truths of physics, the field using the tool, are not (epistemically) necessary. But knowledge of the computational correctness of a step, the tool, is not the same as knowledge of the connection of the rule applied or the formula arrived at with anything epistemically necessary, while logic, the field using the tool, knows necessary truths as such. Still, since logical knowledge is of the necessary while computational knowledge is not, computational methods are to logic as mathematics is to physics: indispensable tools epistemically different from that of which they are the tools. So nothing here demeans the role of computational methods in logic. And although non-standard systems are not interested in the validity of traditional inference forms, still we could not know that their theorems are validated by their rules without implicit knowledge of the validity of the standard inference principles that we must use to draw reach those conclusions.[24]

6.

Having begun above to discuss the epistemic dependence of logical knowledge on the exclusion of contradiction, I will briefly illustrate its dependence on bivalence. Peter Rutz has shown, computationally, that any formula of a multivalent logic is equivalent to an *n*-tuple set of bivalent formulas such that all multivalent theorems are deducible from laws of bivalent logic.[25] But, again, my interest is in precomputational epistemic conditions for doing logic computationally.

Consider the trivalued '->' which is defined to have value .5 when *p* has value 1 and *q* value .5, or *p* has value .5 and *q* value 0. If so, a formula like

(1) ((p -> q) -> p) -> p

would have value .5. But when we know that the trivalued definition assigns (1) the value .5, we also know that the definition either assigns a formula the value .5 or does not assign it .5, and that if a formula does not not have the value .5, it has that value. If we did not know these bivalent truths, we could not understand how any multivalued truth-functional operator is used. Nor is this a case where we can metalinguistically climb a bivalent ladder and then kick the ladder away once we have constructed the multivalent object language. If rules expressed by means of a bivalent metalanguage add the value *X* to the usual 1/0 matrices for evaluating formulas in an object language, the object language will be trivalent. But that does not free us from needing to know the bivalent truth that formulas in the object language either have value *X* or do not have value *X*; and if they have value *X*, they do not not have value *X*. So it remains the case that the logic involved in knowing that a formula in the object language has been assigned a certain value must be standard logic, as must be the logic used in concluding that steps in the object language conform to the rules stated in the metalanguage. Any *truth* we know about multivalent formulas is bivalent.

So the issue here is more basic than whether a metalanguage defining a multivalent object language must be bivalent. To invoke the language/metalanguage distinction as a way to avoid the epistemic dependence of nonstandard methods on standard would be to beg the question of whether knowledge of computational correctness is the same epistemic type as knowledge of logical validity; for the language/metalanguage distinction is relevant to logic only to the extent that it is needed for the use of computational methods, whose relation to knowledge of logical validity is the question at issue. The same kind of epistemic questions arise about an object language, a metalanguage or a meta-metalanguage, questions such as ‘How can we recognize that step 2 in this procedure is justified by step 1 and rule A’, or ‘How do we know that rule B preserves truth’. These are epistemic questions about our knowledge of logical truth. They and their answers are not in a metalanguage, if a ‘metalanguage’ is defined by the *purpose* (or *function,* if ‘purpose’ sounds too psychologistic) of expressing syntactical or semantical rules for another language. From the viewpoint of our precomputational knowledge of necessary truths, it is only incidental that the language in which they are expressed can also be used for that purpose, although it is hardly incidental to our knowledge of computational correctness that precomputational language can also be used for that purpose. But to know inferential validity, we had to have implicit knowledge of necessary truths expressible in precomputational language long before computational systems useful for establishing inferential validity even existed.

And if we use a meta-metalanguage or a meta-meta-metalanguage, some of the same epistemic questions, with the same answers, arise at each level. If the rules expressed in a bivalent meta-metalanguage add the value *Y* to the usual 1/0 matrices for evaluating formulas in the metalanguage, the formulas of the metalanguage will be trivalent, and so the rules expressed in the metalanguage for the object language will be trivalent. Assume that the rules expressed in the trivalent metalanguage make the formulas of the object language similarly trivalent by adding value X. Then, we can know the bivalent truth that formulas in both the metalanguage and the object language either have value *X* or do not have value *X*, respectively; for if they do not not have value *X*, they have value *X*. So, it is not the case that although (a) the logic involved in knowing that steps in a metalanguage conform to the rules expressed in its meta-metalanguage must be standard, still (b) the logic involved in knowing that steps in the subsequent object language conform to the rules of its metalanguage need be no more standard than is the logic expressed by the rules of its metalanguage.

I do not intend to discuss intuitionism (or anti-realism) any more than to note that identifying value 1, say, as 'proven' and value 0 as 'disproven', and even adding value X identified as 'neither proven nor disproven', would not affect the epistemic bivalence that comes from having the use of signs for negation in the ordinary sense. Of any statement we can know either that it is either proven or not proven (which is not to say that it is either proven or disproven), and either disproven or not disproven (which is not to say that it is either disproven or proven). And we can know that it is neither proven nor disproven OR that it is NOT neither proven nor disproven, and we can know that IF it is NOT neither proven or disproven, THEN it is either proven or disproven.

The reply may be that I have not really engaged the opponent because I have changed the rules, for on the basis of their definitions, intuitionists agree that we can know that something is either proven or not proven, since not being proven differs from being disproven. Yes, but in so agreeing the intuitionist is really granting my EPISTEMIC point, independently of whatever kind of point, perhaps metaphysical, he may be trying to make. It is the intuitionist or anti-realist who, from the epistemic point of view, can only succeed in changing the subject, namely, negation and alternation as ordinarily understood and as understood in the context of standard logic, whether traditional or classical modern.

Epistemically, our knowledge of the law of excluded middle is caused by our knowledge of the use that negation signs ordinarily have, the work that we happen to, but need not, give 'not' and '-', the work of applying the relation *other-than* and communicating it. Again, the existence of other kinds of negation is not an issue. Once we have achieved the grasp of ordinary negation, once ordinary negation is part of our conceptual equipment, we are unable not to know, when fully conscious and attentive to the relevant factors, linguistic and/or other than linguistic, that a statement is either proven or not proven, disproven or not disproven, etc. That just happens to be the job that 'not' currently has.

And it does not matter if the use of negation for statements is logically or psychologically prior to its use for predicates. That is a different kind of question. However and whenever we acquire it, once we have negation, as exemplified by the way 'not' is ordinarily used in constructions like 'proven or not proven', in our conceptual equipment, we are stuck with it (fortunately!). And if we have not yet acquired it, we are capable of acquiring it at any time. We can then fail to grasp the truth of certain statements only by not being attentive to something we know, namely, what negation is, since the truth of those statements is caused by what negation is. In other words, we can fail to know those truths only by ignoring the relevant factor. There can be many reasons for ignoring a relevant factor, especially the influence of philosophical problems, questions, mind-sets, cultures and/or theories that appear to be relevant, due to the connectedness and causal complexity of ALL philosophical issues, while they lead us to focus elsewhere than on the relevant factors.

Nothing here detracts from the potential value of studying nonstandard computational systems either for their own sake, for their applications, or for what they can tell us, by way of contrast, about standard logic. But we could not be aware of that value, or of the truth of sentences stating that such systems have a value, if we were not at the same time implicitly aware of the necessary truth of principles of standard logic. Nonstandard systems have value for us only to the extent that our knowledge about them and their applications is consistent and bivalent. Likewise, philosophical critiques of principles of noncontradiction or excluded middle.[26] must rely, explicitly or implicitly, on recognition of the validity of deductive inferences of standard logic. The paraconsistent logician wants to get a conclusion like 'We can avoid the result, $(p^{-}p) \to q$' from the premises 'If disjunctive syllogism is dispensable, we can avoid the result, $(p^{-}p) \to q$' and 'Disjunctive syllogism is dispensable'. But he could not grasp the validity of that inference if he did not rely on implicit knowledge of necessary truths whose necessity derives from the same precomputationally known meaning, namely, negation, that makes disjunctive syllogism necessarily true.

7.

Although computational methods are an indispensable tool for logic, like tools of any kind, they have limitations. I am not just thinking of the fact that each computational method has limits in comparison with other computational methods; for example, Sommers developed a computational system that has advantages over Frege's, while Frege's has some advantages over it. I am thinking of a much more basic limit: Every time we construct a new computational method for verifying logical validity we find that it has characteristics that are, at a minimum, anomalous by the standard of some knowledge that we already possess about what is or is not logically necessary. For example, given the way our society happens to use 'if . . . then', would anyone who did not know modern logic say that the following expresses a valid inference:

1) If (i) there are no unicorns, and (ii) there is no Santa Claus, then (iii) if there are unicorns, there is a Santa Claus.

Even if we find (iii) a good rhetorical device for expressing our skepticism about unicorns, the falsehood of (1) would not require that for which 'if . . . then' is used in this society to be what it is and not be what it is. But the occurrence of such anomalies need not affect either the technical success of truth-functional definitions as a computational method or the usefulness of that method as a tool for verifying necessary logical validity.

Likewise, standard computational methods show the necessary truth-functional validity of a formula like ∃*x*(R*ax* -> R*xb*). Because of that computational validity, Quine calls 'For any two people, John and Mary, there is someone who, if admired by John, admires Mary' *logically true*.[27] But we are able to recognize that this is only an artifact of an (indispensable) *tool* of logic, since we know ∃*x*(R*ax* -> R*xb*) is necessarily true only relative to definitions stipulated by and for the use of that tool, and that under those definitions the formula does not express any precomputationally known necessary truth. For on the basis of that for which we use the vocabulary we call 'logical' in the necessary truths whose knowledge is presupposed by computational knowledge, we can know that the following are possibly true at the same time:

(2) John admires everyone, including John, but Mary; and no one admires Mary.

(3) John's admiring of Mary would cause Mary not to admire Mary.

So, ∃*x*(R*ax* -> R*xb*) cannot express a truth made necessary by the vocabulary we precomputationally call 'logical'. As far as we know, nothing that is made necessary by the way we precomputationally use 'every', 'and', 'not', 'but', 'no' and 'if . . . then . . .' makes it impossible for (2) and (3) to be simultaneously true.

The truth of (3) would not require a concept of causality other than then the one Hume concocted out of desperation, conformity to a universal law. There might be some universal law like "Whenever someone whose first name begins with 'J' admires Mary, Mary does not admire Mary." But for the sake of the record, before children are sophisticated enough to perform the abstractions needed to discover that there are universal laws behind each of the radically individual and unrepeatable concatenations of elements that constitute the contexts of their lives, and so long before they able to be confused by Hume, they believe in causal connections, connections of production or dependence, between what one thing or event is and some other thing or event is. That is, they believe what they could later express by saying that what existent A is can have something do to with what distinct existent B is. And they believe that there are necessary causal connections, since they believe that no change would exist without the existence of something other than itself. We, who are just those children at a later age, can express that necessity by saying that if A exists and B does not, then at least one of A and B both is what it is and is not what it is. If postulating such a causal connection between John's admiring Mary and Mary's not admiring Mary violated any precomputationally known logical necessity, we could not, not just would not, have any beliefs about causal connections between what one state of affairs in and another state of affairs is.[28]

C. I. Lewis was right that the necessity of B's being true if A -> B and A are true is included in our precomputational understanding that for which 'if . . . then' is used (whether or not we explicitly express that necessity with modal vocabulary). But it does not follow that a computational modal logic can capture that for which we use 'if . . . then' in a way that does not generate anomalies by the standard of precomputationally known necessary truth. Because the prior grasp of necessary truth on which the grasp of computational correctness depends is non-computational, computational methods will always be anomalous if considered to be models of how we know necessary truths of logic. Since knowledge of computational correctness presupposes knowledge gained noncomputationally, there must always be something lacking in computational methods *if they are considered as models of what they PRESUPPOSE* and which they therefore cannot contain. But it does not follow that they lack anything considered in themselves.

Consider another kind of anomaly. T. H. Irwin finds Aristotle's account of valid deduction 'relatively narrow' as compared to what is now 'a more familiar' account: 'An argument may be valid even if it is redundant, or a premise is identical to the conclusion, or it has only one premise . . .'[29] But no one would consider *p* -> *p*, for example, a deduction — and much less a valid deduction — on the basis of their implicit precomputational grasp of what valid inference is. It is only after learning modern propositional logic and being justifiably impressed with the unsurpassable power of computational methods of verifying validity, that we conclude that p -> p should be considered an inference because it meets the tests for being a wff and a tautology in the best method we have for determining validity. And there is nothing wrong with the propositional calculus because it puts p -> p on a par with ((p -> q)p) -> q. That is not an error any more than material implication is an error. It is just another 'limitation', from the point of view of our pre-computational grasp of valid inference, characterizing this tool for achieving explicit knowledge of logical necessity. Such anomalies are a small price to pay indeed considering (1) that any tool in any kind of endeavor will have limitations and (2) the power that this tool gives us.

To illustrate the point that *any* computational tool for verifying noncomputationally known necessary truths will have limitations, consider the system by which Fred Sommers has shown, against the previously all but universal view, that the traditional 'two term' analysis of statements can validate all the inference forms that Frege's function/argument analysis can validate. Sommer's method succeeds only by introducing its own anomalies (by the pre-computational standards of 'The Logic of "Ordinary Language"' that Sommers is trying to give us). And, again, there is nothing wrong with that. For example, he shows that two-term categorical syllogisms can handle the relational statements that only Fregean polyadic predicates are supposed to be able to handle. To do so he, following Leibniz, uses redundant premises like 'Every R to an X is R to an X'.[30] But by the standards of precomputational knowledge of inferential validity, using a redundant premise is no less anomalous — and no more — than counting p -> p as an inference.

Sometimes we describe our precomputational grasp that a logical truth is necessary as 'intuitive'. So the anomalies we always find in computational methods are judged to be so by the standard of pre-computational intuitions. But a common implication of the word 'intuition' is that intuitions can result in false beliefs. We want computational methods to replace reliance on intuition in the sense of reliance on something that is not reliable. However, they cannot replace but depend on actual pre-computational knowledge that certain logical truths are necessarily true. So some things that are anomalous by the standard of logical truths whose necessity we must know, rather than just believe, will always characterize computational methods for logical verification.

In order to do predicate logic 'semantically', that is, by using so-called 'truth'-conditions as the bases of rules and definitions, Tarski invented a nonself-referential language in which contradictions of the self-referential kind that we know and love so well are not possible (and note that we need to *know* their impossibility to know this advantage of Tarski's method; so their non-occurrence in the language has to be epistemically necessary). The irony is that the purpose for 'true' in precomputational language is precisely linguistic self-reference. 'True' is meant to be applicable to other linguistically generated structures. And from an epistemic standpoint that linguistic self-reference is not accidental to that for which we use 'true', hence my quotation marks for Tarski's computational 'truth'-conditions. Even if we added 'true' to a language that did not have it merely for the Ramseyan purpose of assertive redundancy, we would be doing so for a linguistically self-referential purpose.

But though a Ramseyan 'true' could be eliminated as redundant in certain contexts, even in those contexts one side of our belief in "It is snowing' must be 'self-referential'. That is, when we believe a statement like 'It is snowing' because we see the snow, we have a two-sided consciousness. We are conscious of something extra-linguistic, the weather. And we are conscious of the linguistic fact that the weather is successfully articulated in 'It is snowing'. Our assent to 'It is snowing' is not just a conscious relation to what it other than ourselves, the weather. It is also an at least implicit conscious relation to something about ourselves, the fact that we have formulated a statement about the weather. So only a language with self-referential capability can adequately express our epistemic situation. As a result, it has been recognized that for epistemic questions we would have to substitute for truth other linguistically self-referential properties of sentences like 'should be believed' or 'is justifiably believed'. And when we have beliefs about the correctness of steps in a Tarskian method, some linguistically self-referential properties like that would have to characterize those beliefs, just as consistency and bivalence must characterize them.

But that does not internally tarnish Tarskian semantics one bit. Nor does it mean that proof-theoretic approaches to computational verification of validity have more claim than semantic approaches to being legitimate offspring of, or models of, our precomputational grasp of validity. Any computational approach to logical consequence must have limitations with respect to resembling the knowledge of logical validity its use presupposes.

One of the most well known limitations of computational tools is, of course, that no (consistent and bivalent) computational system adequate for expressing arithmetic can prove all the truths expressible in the system. But my point about the inevitable limitations of computational methods comes from a more epistemically fundamental perspective than Gödel's. My point is from the precomputational perspective of the epistemic conditions for the knowledge of necessary logical truth that is needed for grasping computational correctness, while Gödel's point is from a perspective internal to computational methods, the methods whose use for acquiring knowledge presupposes the conditions I am examining. As such, my point about the limits of computational methods concerns computationally proven and provable, not unproven or unprovable, truths. To know that a computational process proves anything, we must know necessary truths of logic otherwise than by computational methods.

This point is closer to Putnam's demonstration that how we know a proof to be 'sound' (meaning logically valid with true premises) cannot be formalized in a system, including a formal system of inductive logic, that we could recognize, computationally, as itself sound without running up against Gödel. From there, Putnam concludes that '*reason can go beyond whatever reason can formalize*' (his emphasis).[31] As well as being more fundamental, I am trying to be more specific by showing that one way reason *must* go beyond what it can formalize is by noncomputationally knowing necessary truths presupposed by our ability to recognize success in formalization.

Also, machines can arrive at correct computational results. But since current (and all future?) machines are so designed that their abilities are restricted to achieving what can be achieved by computational methods, machines are not capable of the kind of recognition we achieve in the noncomputational knowledge on which knowledge of computational correctness depends. The inability of machines to achieve the kind of knowledge of the necessary that we have is itself a (causally) necessary consequence the fact that knowledge of computational correctness presupposes knowledge gained precomputationally.

Certainly, we have sufficient evidence that some animals can have a kind of knowledge that a rule they have learned has been violated. They demonstrate that by their negative reactions if a behavioral expectation we have given them in teaching them a rule, for example, that such and such behavior will get such and such a reward, is frustrated. But that is not evidence that they grasp the logical relation of necessarily valid inference. Expecting that an event will occur is one thing; knowing (certitude caused by awareness of evidence sufficient to exclude the opposite from truth) that a statement asserting that the occurrence will occur is true is another. Likewise, many higher animals can acquire an expectation based on the repeated connection of two kinds of events; that is not the same as inductively concluding from past evidence that it is unreasonable to believe the opposite of a universal statement asserting that connection. And animals can use words with universal meanings, but that gives us no more evidence that they can recognize the logical relation of universality than we have that they can recognize the logical relation of implication or the epistemological relation of justifiable belief. To show that animals can have an even *implicit* knowledge that an inference is valid, we would need evidence that they can have explicit knowledge that sentences making up an inference are simultaneously true. For it is in knowing or hypothesizing the truth of sentences, that we have implicit knowledge of the necessary validity of inference forms.

Interestingly, these caveats about machines and animals do not directly disprove materialism in the philosophy of mind; they just disprove certain arguments for materialism. But what if it turned out to be the case that the methods of empirical science are restricted to giving information about processes that can be represented by computational models? As we know, the methods of science are restricted to giving us information about certain fundamental things only statistically, not deterministically. Likewise, those methods might be restricted to only giving us information expressible in algorithms of physical processes. In that case, scientific theory could cover every scientifically knowable piece of empirical data about how our brain functions in recognizing logical necessity while telling us *nothing* about how we noncomputationally acquire the grasp of inferential validity that judging inference forms to be valid by the correctness of computational steps presupposes. Would this scientific inability disprove materialism? No, it would only be further evidence for the limitations of scientific methods, something we know already from science's inability to know the speed and location of a particle at the same time or the simultaneity (which we know to hold; see Chapter 8 of *Causal Realism*) of distant events. (If we can accept Gödel's proofs of the limitations of computational methods for mathematics, we should be able to accept science's revelations about its own limitations.)

Part II

8.

Generally, principles of valid inference are not truths that are necessary in the sense that their direct denials have the form 'A and not A'. Modus ponens is necessarily true not because its denial is the statement 'What implication is is not what implication is', and the denial of dictum de omni is not the statement 'That for which "all" is used is what it is and is not what it is'. Rather such principles are epistemically necessary only because we are capable of recognizing that their falsehood would depend on such contradictions being true. How do we recognize that? Behind that question about knowledge is a question about truth: How can the truth of a statement whose opposite is not a claim that something is and is not what it is depend on the falsehood of such a claim? In other words, how can a statement whose opposite is not a direct contradiction be a necessary truth? I will answer the question about truth first and then derive the answer to the question about knowledge of truth from it.

I will start by answering both questions for a kind of necessary truth not ordinarily considered 'logical,' then apply the answers to the kind traditionally called 'logical,' and then to the kind specifically studied in the science called 'logic.' Starting from a broader concept of necessary truth will show that this theory of necessity is not an ad hoc construct designed to save a theory of logic. Each kind of necessary truth derives from conditions natural to human cognition and language.

The same prelinguistic object of experience may have features such as being a toe and being red. On *any* analysis of features, what it is to be a toe and what it is to be something red are prelinguistically diverse. Their diversity does not derive solely from language; for being a toe and being red can occur separately from each other. But if 'My toe is red' is true, the same prelinguistic object of experience that sentence succeeds in describing by 'toe' it succeeds in describing by 'red'. So the terms of the diverse linguistic relations, describable-by-'toe' and describable-by-'red', do not differ in their prelinguistic state. The prelinguistic sameness of these terms is contingent, however; for we accomplish the feat of diversely describing this nondiverse thing by mentioning features of the thing (being a toe and being red) that are prelinguistically diverse and are contingently connected in their prelinguistic state.

Now consider how we manage to describe the same thing diversely in 'A red thing is a colored thing'. What we are aware of in knowing that for which 'red' , as a noun or an adjective, is used differs from what we are aware of in knowing that for which 'color' or 'colored', respectively, are used. But assume the following temporarily, for the sake of argument:

(I) Any information communicated by 'color' or 'colored' is communicated by 'red' (as a noun or adjective, respectively), but 'red' communicates more information than they do.

If so, whatever object of experience we can succeed in describing by the adjective 'red', we can succeed in describing by the adjective 'colored', but not vice versa. In contrast, consider the noun forms 'red' and 'color of a toe'. Whoever has the information communicated by the first does not automatically have the information communicated by the second, even though 'red' and 'color of a toe' may describe the same object of experience.

Now make one more temporary assumption:

(Ia) If (I) is true of 'red' and 'a color' but not of 'red' and 'color of a toe', at least part of the reason is that the only differences between what we are aware of in knowing that for which 'red' and 'a color' are used are differences on the side of our manner of articulating experience in language to the exclusion of being differences on the side of the prelinguistic feature of experience that is articulated.

What it is to be a color does not differ from what it is to be red (or any other color) as one feature of experience differs from another prior to language. A thing is not made red by one feature and colored by another.

Quantifying over 'features' will not be an issue here; for these temporary assumptions will become dispensable (in Section 14) after serving a heuristic purpose. But the view that diversity in predicates must reflect a diversity of features is so common as to require some comment. That view makes the number of a thing's prelinguistic features depend on the resources of the language describing it. And that view multiplies causes of diversity in meaning far more (infinitely so) than necessary, since the diversities in question can be accounted for by the abilities of the describer independently of what she is describing. If each of 'crimson', 'red', 'chromatic color', 'color' requires a distinct feature, then instead of the state of affairs, p, being sufficient to *cause* not only *'p'* to be true but also '*p* or *q'*, '*p* or *q* or *r'*, '*p* or *q* or *r* or *s*', etc. to be true, there should be a different state of affairs making each sentence in this infinite set true.

The perceived correlation between diversity of meanings and diversity of features may come from the view that sentential truth requires there to be an equivalence between some 'logical' characteristics of language and the extralogical reality that is expressed in language. The *Tractatus'* theory of truth was such a view, but that view does not need to be associated with a picture theory of truth. The present analysis will show that precisely the opposite of that view is the case: truth requires that what is shared by language and reality is not a 'logical' property of language in any meaning of that word that will be relevant to this discussion. Another source of the perceived correlation between distinct predicates and distinct features may come from the fact that computational methods for 'second-order' logic can stipulate distinct existential quantifications over predicates for every difference in the extension of predicates. I will argue (in Part III) that logical rules for existential quantification need have *nothing* to do with metaphysical questions about what exists, and *not* because 'exists' has multiple *logical* meanings; it does not. But where there is a diversity in the logical properties of two words for features, there need not be an ontologically distinct feature in a given individual of which both words can be predicated.

The second temporary assumption (Ia), then, could also be put this way: Since the information communicated by 'red' and 'color' is the same except that 'red' communicates more than does 'color', what we are aware of in understanding how 'colored' is used is just more vague, abstract or generic, and less precise, detailed or specific, than what we are aware of in understanding how 'red' is used. ('Vague' here refers to a relation between the uses of different general terms, not between the uses of general terms and individuals.) Being more- vague-than, less-precise-than, etc. are characteristics of that for which we use 'color', namely, color, relative to that for which we use 'red', namely, the color red; for they are relations, not between the noises 'color' and 'red', nor between each of those noises and the respective information they communicate, but between the information communicated by 'color' and the information communicated by 'red'. But compare relations such as being more or less vague (precise, abstract, etc.) to features such as being a toe or being colored. The former are relations belonging to objects of cognition *as a result of those objects' being expressed in language*; I will call those relations 'linguistically generated' objects of cognition (LGOs).[32] The latter are features that can belong to an object of cognition prelinguistically, and so belong to it other than as a result of its being expressed in language.

When language occurs, prelinguistic objects become associated with properties and relations that are causally dependent on language, LGOs. For example, were it not for language, it would not be true of Cicero that he is called 'Cicero'. A more sophisticated example: Before language there were more material things than vegetative things and more stars than cats. But only when language exists do 'material' and 'star' have greater *extension*, respectively, than 'vegetative' and 'cat'. Extension is an LGO, a relation that comes into cognition only because of the way we use words. So long as our cognitions and linguistic articulations of cognition are *fragmentary, limited, multiple, more detailed and less detailed*, and so on, when aspects of prelinguistic experience go from not being articulated in language to being so articulated, they become associated with cognizable properties and relations that would not themselves be objects we are aware of had prelinguistic features not become that for which we use certain noises and shapes as signs.

Here a digression on some terminology to be used throughout Part II is called for. 'Object' means a term of some cognitional relation, a *relatum* that is cognized in some way such as being perceived, understood, conceived, meant, referred to, named, described, etc.

'Object' is used in contrast with 'thing,' which means an actual or possible extracognitional existent (a real existent where "really existing" is opposed to merely being imagined or conceived. 'Extra' does not imply 'beyond' or 'outside of' cognition in the sense that what is unknown is outside of cognition. For the arguments to follow, it is sufficient for existing 'extracognitionally' to mean having a status that is *more than* the status of *merely* being conceived or imagined or in some other way being merely cognized.) What is an object can also be a thing or be a merely cognition-constituted object, an object that can have no status other than being imagined, conceived. 'To objectify' and 'objectification' refer to the coming about of something, either a thing or something cognition-constituted, being truthfully describable as an 'object,' that is, as known, mentioned, named, seen, etc.

In some important way, "secondary" sensory qualities are subjective, and to that extent they are cognition-constituted objects. But that cannot be the whole story. Wittgenstein's private language argument tells us more than that language initially occurs for the sake of communicating about public objects as opposed to private subjective objects like pains; *Remarks on the Foundations of Mathematics* indicates that language occurs to communicate about objects that are public in the sense of existing extracognitionally, as opposed to being merely imaginary.[33] The initial objects of whatever kind of cognition is necessary for language must be cognized as possibly public, and so extracognitionally existing, things. Since 'Everything in the intellect comes through the senses,' the way in which sensory qualities are subjective must be consistent with their making us directly, that is, noninferentially, aware of real public existents. We could not initially infer public extra-objective existents, because we must use language to infer, and language must start with concepts about them, not about subjective cognition-constituted objects. I have elsewhere shown how the subjectivity of sensory qualities is not only compatible with direct awareness of extra-objective existents but is needed for it.[34] Here, the only cognition-constituted objects I will need to discuss are associated with the kind of cognition, whatever that may be, posterior to sense perception that is required for language. So to avoid the appearance of raising questions that apply to sensory qualities, I will generally use the term LGO where I could have used cognition-constituted object. The point of the arguments will not be affected.

A reader, however, might not find 'LGO' broad enough for some cognition-constituted objects. For example, did human beings need to develop language to cognize negation, the relation other-than; do babies need language before grasping that one thing is different-from another? So perhaps stipulating a richer vocabulary would be helpful here dialectically and pedagogically, though the validity of the arguments would not be depend on it. But perhaps a richer vocabulary would not help. For example, the search for necessary and sufficient criteria for the use of words is usually an obstacle, not an aid, to solving philosophical problems. In fact, a main point of the arguments to follow will be to show that necessary and sufficient criteria for the use of words is usually irrelevant to the solution of philosophical problems. Particularly irrelevant to the necessity of truths that we happen to usually call truths of 'logic,' and to our knowledge of them, would be necessary and sufficient criteria for words like 'logical,' as in 'logical particles,' or 'synonymy,' as related to 'analytic' truths, or even 'linguistic' and 'cognition.' The only *necessary* condition for the explanation of a term in a philosophical argument is that the explanation be *sufficient* for the specific use of the term in the specific argument being considered, even, and especially, when the argument concerns objects like *explanation of a term* or *philosophical argument*. By *causal* necessity, what conditions are *sufficient* for the explanation of a term in a philosophical argument will vary with the purpose of the argument and a term's role as a means toward that purpose.

To return to 'Red is a color': The second assumption (Ia) means that if the first assumption (I) is true of 'red' and 'a color' but not of 'red' and 'color of a toe', at least part of the reason is as follows:

(II) The only differences between what we are aware of in knowing that for which 'red' and 'a color' are used are LGOs, differences on the side of our manner of articulating experience to the exclusion of being differences on the side of that which is articulated.

The difference between the objects of cognition *red* and *color* are cognition-constituted relations, like being-more-precise-than, only. On the other hand, the differences between what we are aware of when we know that for which 'red' and 'color of a toe' are used do not consist solely of properties resulting from our manner of expressing things. Understanding that for which we use 'color of a toe' requires an understanding of what a toe, is; a toe is not LGO. So to understand that for which 'color of a toe' is used we have to be aware of something that would differ from the color red even if there were no cognition or language.

LGO is a psychological concept, so is cognition. But LGOs like extension and precision are not psychological relations. If we need psychological acts, states or mechanism to relate to objects cognitionally, linguistically generated relations are among the objects we relate to cognitionally, not among the psychological conditions needed for cognition. Linguistically generated relations are objects that come into cognition (a) *as attributable to that for which we use language*, attributable to the information communicated, not as attributable to any mental mechanism that may or may not be behind our use of language. Perhaps, for example, colors have no status independent of our perceptions. But it is the prelinguistic perceptual information conveyed by 'color' that is contained in that conveyed by 'red', not information about whatever mental states beyond perception may or may not be required to articulate perceptual information in language. And linguistically generated relations come into cognition (b) *as a result of our use of language*, whatever mental mechanisms may or may not be behind our use of language. When epistemology uses psychological concepts, it uses them to analyze our cognition of nonpsychological objects, including the linguistically generated relations that, I will argue, can cause the necessity of 'logical' truths.[35] So this account of logical necessity will not be psychologistic. (I will further specify the meaning of 'logical' sufficiently for the purposes of these arguments in Section 14.)

If the subtly of this explanation of why radically different kinds of properties can each pertain to that about which we use language, as well as the distinction between lexicological and nonlexicological understanding of usage, is somewhat annoying, only the need for subtlety like that can explain the much more than annoying fact that the philosophy of logic has not found an alternative to the laws-of-thought/laws-of-'abstract'-objects false dichotomy. And only that need can explain the constancy of the temptation to turn logic into metaphysical knowledge about how objects exist cognition-independently.[36]

Assumptions (I) and (II) will now allow us to implement the unheard of idea of explaining necessary truth, like that of 'Anything red is colored,' *as a function of what makes statements in general true*, whether necessarily true or contingently true.

9.

There is affirmative truth when what is said (and thus is objectified in cognition) is the same as what exists (extracognitionally). Leaving aside the question of single-word truths, affirmative truth follows from the identity of diverse cognitional objects, the terms of diverse relations of naming, describing, predicating, referring, etc. with the same actual or possible extracognitional thing. All affirmative truths,[37] necessary or contingent, that diversely express the same thing are made true by the fact that the terms of diverse linguistic relations are not diverse otherwise than as terms of linguistic relations. They are true because the terms of diverse relations like successfully-named-by-'Al' and successfully-described-by-'*F*', or successfully-described-by-'*F*' and successfully-described-by-'*G,*' are not diverse but identical in what they are prelinguistically, not diverse but identical insofar as they have a status of being *more-than-just-terms* of linguistic relations. Although I believe this condition for truth implies an account of what makes attempts to name and describe successful, we do not need such an account here. In each of 'My toe is red', 'A red thing is a colored thing', and 'Red is a color', we describe something diversely that is prelinguistically nondiverse. The thing 'toe' here describes is the same thing 'red' describes; anything that 'red' describes is the same as something 'colored' describes. So the question of whether 'My toe is red' and 'A red thing is a colored thing' are necessarily true is the question of whether, without contradiction, 'toe' and 'red' are each able to successfully describe anything that the other does not, and 'red' is able to successfully describe anything that 'color' does not — whatever it may be that constitutes the success of any of these descriptions *taken individually*. (Unless otherwise noted, I will henceforth assume without mentioning it that the naming and describing in the examples are successful.)

The differences between the ways that what is prelinguistically the same becomes, as a result of human behavior, a term of the diverse linguistic relations, describable-by-'toe' and describable-by-'red', in one case, and describable-by-'red' and describable-by-'colored', in the other, have different *effects*. The differences between the ways a prelinguistic object becomes the term of the diverse relations, describable-by-'toe' and describable-by-'red', make 'toe' able to noncontradictorily express something other than what 'red' can express. The differences between the ways a prelinguistic object becomes the term of the diverse relations, describable-by-'red' and describable-by-'colored', make 'red' unable to noncontradictorily express something other than what 'colored' can express. In the first case, the diversity in the descriptions consists of their describing *by means of features that are prelinguistically distinct* such that either of them can exist without the other. So the prelinguistic identity of what is described by 'toe' and by 'red' is contingent. 'Toe' is able to be accurately predicated of something that 'red' is not, and 'My toe is red' is contingently true.[38]

In the other case, since the diversity in the meanings of the descriptions consists solely of LGOs, the diversity comes solely from the abilities of the describer in a way that excludes any prelinguistic distinction; so the ways we happen to use 'red' and 'colored' as means of articulating prelinguistic experience make 'red' incapable of being accurately predicated of anything that 'colored' cannot. On hypothesis (II), namely, that the diversity between that for which we happen to use 'red' (namely, what red is) and 'colored' (what color is) is linguistically generated *only*, it would be *contradictory* for any instance of what red prelinguistically is not to an instance of what color prelinguistically is. For if what it is that is the term of the relation describable-by-'red' was *prelinguistically* diverse from what it is that is the term of the relation describable-by-'color', then either what red or color, or both, is hypothesized to be is also not what red or color or both is hypothesized to be, namely, something that is not prelinguistically distinct from the other.

The nature of the diversity between that for which we use 'red' and 'color' makes 'Red is a color' or 'Anything red is colored' true on pain of contradiction. For given that for which the rest of the words in hypothesis (II) are used, on the assumption that (II) is true that for which we happen to use 'red' is related to that for which we happen to use 'color' solely by a linguistically generated feature, being-less-precise-than, such that there can be no prelinguistic difference between what being red is and being a color is, by hypothesis. As long as the cause of their diversity in meaning prevents 'red' from describing anything that is not prelinguistically the same as what 'a color' describes, red can fail to be a color if and only if the prelinguistic feature, red, and/or the prelinguistic feature, color, is and is not what it is. So 'Red is a color' can fail to be true only if that for which we use 'red' and/or 'color' is and is not what it is, that is, only if the color red is not the color red or color is not color.

This point is so central to epistemology[39] that I will put it one more way, using the thing/object terminology. Since affirmative truth follows from the identity of certain diverse objects of cognition with the same actual or possible extracognitional things, a truth is necessary if and only if the extracognitional identity of these cognitionally diverse objects is necessary. So the question is why must what are diverse as objects of cognition not be diverse as cognition-independent things, not be diverse otherwise than as objects. If the diversity is not on the side of what pertains to things as things, it depends (causally) on the way things are diversified as objects, on the way we manage to diversely objectify things. It is possible to so objectify things that what are diverse as objects could only be diverse as objects, not as things, on pain of contradiction. How? One way is by diversifying objects solely by properties that pertain to the objects as objects of diverse cognitional relations only, not as cognition-independent things. (Another way is by diversifying objectifications of the same thing by alluding to extracognitionally diverse features, like being a color and being in a place, that are related by causal necessity, like the formal/material causal relation between color and place.) If the diversity between objects consists only of properties belonging to the objects in their state of being objects, like the diversity between what red is and what color is, the way we have diversely objectified something leaves no room for a diversity pertaining to what is objectified insofar as it is actually or possibly more-than-an-object. A difference between the objects, what red is and what color is, insofar as they are extracognitional values would contradict the hypothesis that the way we have diversified these objects causes their difference to consist solely of properties coming to them from cognition.

If the relations between the meanings of predicate 'F and predicate 'G' are such that the difference between these meanings is only a cognition-constituted difference, these words amount only to different ways of articulating the same extracognitional value. Again, the issue is not whether the relations between 'red' and 'color' and that for which we use 'red' and 'color, respectively, are what they are but whether red and color, which happen to be that for which we use 'red' and 'color,' are what they are. On the other hand, the uses that the linguistic tools, 'toe' and 'red', happen to have make these tools capable of describing prelinguistic objects that are only contingently the same; for the cause of the diversity between the ways we use 'a toe' and 'something red' is the fact that they describe by means of features that are prelinguistically separable. (Nor could any 'incommensurability' of meanings, caused by a change in some belief, affect the necessity of 'Red is a color'. As long as its words mean what they *now* do, if 'Red is a color' were not true, one or both of the prelinguistic values red or color would be and not be what it prelinguistically is. Besides, incommensurable meanings for a word cannot prevent beliefs employing the word from being simultaneously true; for sentences cannot contradict each other unless they use words in the *same* way.[40])

This does not explain how we *know* that sentences like 'Red is a color' are necessarily true; rather it explains why they are necessarily *true*, that is, why the term of the relation describable-by-'red' must be something that is also the term of the relation describable-by-'a color'. But since what red is is not the same as what the relation between red and the noise 'red' is, we should not call this a 'linguistic' theory of necessity. Necessary *truth* is no more relative to language than is contingent truth, although (with caveats to be discussed next) *knowledge* of necessary truth is, due to the causal relation between that knowledge and language. With some of the same caveats, we can say that 'Red is a color' is 'true by meaning', in a nonlexicological sense of 'meaning'. But red is not the same as a color linguistically; it is other than a color linguistically. For being-expressed-by-'red' is other than being-expressed-by-'a color', though the otherness between that for which 'red' and 'a color' are used is linguistic otherness to the point of being exclusively linguistically generated.

The *cause* of the terms of the relations, expressed-by-'. . .', being extra-expressedly the same (the sole truth condition relevant here for statements, whether necessarily or contingently true, that multiply describe the same thing) is able to consist solely of linguistically generated, as opposed to prelinguistic, characteristics of that for which we use means of expression. But *at the origin of language, means of expression are intended as means of expressing what things prelinguistically are*. Red and a color are the same in what red and a color are prelinguistically; and 'Red is a color' expresses something about what red prelinguistically is, not what red is only as a result of being articulated in language.

Before moving to our ability to know necessary truth, a few more comments about the analysis of truth used to explain necessary truth in terms of what makes sentences true will prevent some possible misunderstandings. Truth presupposes the prelinguistic identity of the terms of diverse linguistic relations. But the 'pre' in 'prelinguistic' means with respect to the specific relations of named-by and/or described-by in question. So even the truth of 'Precision is an LGO' would require an identity that is prelinguistic in this sense, despite the fact that for which we use both 'precision' and 'LGO' are linguistically generated. (See Section ??) Language first comes into existence, however, for the sake of communicating about the public objects of prelinguistic experience. So in addition to being terms of linguistic relations, the terms of our primary linguistic relations are things that are prior to language altogether, altogether prelinguistic. That is, the terms of the relations required for the truth of 'The toe is red', must like all truths be 'pre'-linguistic with respect to the specific linguistic relations, described-by- 'toe' and by-'red', but must also be prelinguistic in the foundational sense that what we originally and directly recognize to be a toe and red we can later and reflexively recognize to be 'prelinguistic'. (I will apply this analysis of truth to statements about LGOs where appropriate. Unless otherwise noted the discussion will deal with truths about prelinguistic experience, to avoid complicating things more than necessary.)

We can call this an 'identity' theory of truth. But truth does not consist, as Frege thought, of an identity between a statement and a fact or state of affairs. The identity that causes truth is not between a statement and prelinguistic reality. It is identity between the terms of a statement's diverse relations of named-by and/or described-by and the same prelinguistic reality. Since to be described-by-'F' and described-by-'G' is to be linguistically distinct, in order for the sentence 'An F is a G' to be characterized by the relation, truth, there must be a prior relation of identity between the term of the linguistic relation, described-by-'F', and something that is at least potentially more than just the term of that relation; likewise for the relation, described-by-'G'. If those relata were only what is expressed by each of 'F' and 'G', they could never be identical without ceasing to be what is expressed by one of these predicates. A cognitional object that is term of either relation must be capable of being, and known to be capable of being, identical with something that is more than the term of that relation; otherwise, it could neither be, nor be known to be, also the term of the other relation. For a sentence like 'aRbc' to be true, the terms of the relations, named-by-'a', 'b' and 'c' and described-by-'. . . R . . .' must each be identical with something known to be capable of being more than the terms of those relations. So sentential truth requires at least two epistemically prior relations of identity between, on the one hand, the terms of diverse presentential linguistic relations, like being named or described, and on the other hand, something that is more than just the term of each of those relations. The foundation making truth possible is the prior identity of what is expressed by each name or predicate with something that has the potential for being prelinguistically more than what is so expressed.

That is why the identity required for truth cannot consist of identity between reality and some 'logical' (i.e., cognition-constituted) characteristic of either statements, names or predicates. At the foundation of language, truth requires that the term of the identity relation that causes truth is something prelinguistic, not something linguistically generated. So what is shared by language and reality must be some prelinguistic value, like that for which 'toe', or 'red', or 'color' is used. For a cognitional object like what it is to a toe, or red, or a color is both something apprehended by means of prelinguistic characteristics and something that is apprehendable as term of linguistically generated relations once it is expressed in language. But it is the respect, or the respects, in which what we are here describing as 'expressed in language' is identical with something that is also describable as 'more than just what is expressed by the linguistic tool in question' that permits something to also be identical with what is expressed in some other way. Epistemically a toe can be looked at as something prelinguistic or as something described in language. But it is not as what is successfully *described* as a toe that something is identical with what is successfully *described* as red. It is as something whose prelinguistic characteristics allow it to be successfully described as a toe and at the same time make it potentially describable in other ways, because they are prelinguistic characteristics, that the same thing can be successfully described as red.

Whatever else 'logical' characteristics may be, the logical characteristics relevant to our epistemic purposes are LGOs–or at least cognition-constituted objects–and so cannot have the identity with something prelinguistic–or extracognitional–that truth requires. The prelinguistic cause of the truth of 'That is a toe' is the same as the prelinguistic cause of the truth of 'That is a bodily appendage'. The prelinguistic cause of the truth of 'That is red' is the same as the prelinguistic cause of the truth of 'That is colored'.

And this identity theory of truth has nothing to do with confusing the copula with the 'is' of identity. As articulated by 'Al' and 'an F' in 'Al is an F', something is diverse, not identical; it is diverse as the term of distinct relations of articulation. The truth of the copula in 'Al is an F' depends on the prior presentential identities of the term of the relation, named-by-'Al', with something prelinguistic and of the term of the distinct relation, described-by-F, with something prelinguistic. Those prior identities are equally required when we express the truth that 'Al is an F' expresses without using 'is', as in 'Al has F-ness' or 'The toe has redness'. The term of the relation, described-by-'has F-ness' must be identical with something capable of being more than just what is described-by-'has F-ness'. Of course, we probably can't express the relation, identity, in language until after we have words like the copula, but that is the case for many things expressed in a language like ours that relies so much on the copula. But once we have the word, 'identity', we are capable of recognizing that it expresses a relation (which we even later can recognize to be an LGO) that had to hold between the linguistic and the prelinguistic prior to our ability to have the word, 'identity', or to use the copula.

10.

How can we *know* the necessary truth of 'Red is a color' if the necessity of that truth is caused in the manner described? It may seem that my explanation of necessity implies that, to recognize that this sentence is necessarily true, we need criteria for identifying instances in which word usage so differs by LGOs that the linguistically diverse are excluded from being prelinguistically diverse. And the problem of finding criteria for such purposes is Quine's objection to 'analytic' truth.

He asked whether a sentence like 'No bachelor is married' is analytic because it can be turned into a 'logical' truth by replacing synonyms with synonyms. A logical truth would be one that remains true under all reinterpretations of its components but for the 'logical" particles, for example, 'No unmarried man is married'. But by what criteria can we decide 'synonymy' for the purpose of identifying analytic truths? And how do we distinguish particles that are logical from those that are not, short of putting forward an arbitrary list of signs and stipulating that we will use the noise 'logical' for them? Or is 'No bachelor is married' analytic because it is true by virtue of meaning? The word 'meaning' does not help us to find criteria; it is 'as mysterious (and in the same way) as what we want to define'.[41]

Nothing could be more correct. But if there are sentences true by virtue of meaning, recognizing their truth cannot be the result of employing a criterion for identifying sentences true by virtue of meaning. And the preceding sentence is itself knowably necessary 'by virtue of meaning'.

The traditional name for truths knowable solely from an understanding of the way their words are used is 'self-evident' (*per se nota*) truths. The concept of an analytic truth as true solely by virtue of the way its words are used probably derives from the earlier concept of self-evident truth, with two important differences. First, 'analytic' as here defined refers to what makes some sentences true; 'self-evidence' refers to what makes some truths knowable to us. Of course, if a sentence is true by virtue of the way its words are used, its truth should be knowable by knowing how its words are used. But, second, analytic truths would include self-evident truths and truths knowable by reasoning from self-evident truths. Derived analytic truths would not be knowable solely by understanding how their own words, rather than other words in the truths from which they are derived, are used.

To know any necessary truth, we must first know self-evidently necessary truths, truths whose opposites are self-evidently contradictory; otherwise we could not know any derived necessary truths. To know that the opposite of a self-evidently true sentence is contradictory, all that can be required is an understanding of the way its words are used. For example, it is self-evident that it would be contradictory for a sentence to be known solely from an understanding of its words and at the same time to be known by means of some other previously known sentence, such as a sentence expressing a criterion for identifying instances of self-evidence.

Knowledge of the ways we use words can cause knowledge of *prelinguistic* truth, because the relevant knowledge of usage is not lexicological. If someone thought that we use 'soft' and 'sweet' the way we use 'red' and 'colored', respectively, he could know the same truth we express by 'What is red is colored' by being aware of what he thinks we express by 'What is soft is sweet'. The knowledge required is not knowledge of what the relation is between 'red' and that for which we happen to use 'red'; it is an awareness of what the color red is sufficient to make red that for which we use 'red '. Of course, we do not learn any word separately from others, but the question of how we learn contingent lexicological truths about usage is different from the question how we grasp self-evident truth. The fact that we depend on collateral knowledge for learning how a sentence's words are used does not mean that we depend on inference from premises to know the sentence's truth. In traditional vocabulary, the order of discovery is not the same as the order of verification.

Hence, the issue of synonymy, over which so much ink has been spilt, is irrelevant to our knowledge of 'analytic' truths. Knowledge of synonymy is the lexicological knowledge that certain noises do indeed have a certain use; it is not the knowledge of that for which a noise is used that a person could have while being mistaken about which noise has that use. So I am not defending knowledge of 'truth by convention'.[42] Stipulation is not what makes truths self-evident. Stipulation produces only a lexicological relation. Truths can be self-evident due to that which we stipulate, but the self-evident necessity comes from other LGOs holding between the stipulated meanings, not from the stipulation itself.

The noises 'Cicero' and 'Tully' are related only by their stipulation for the same individual. It is self-evidently necessary that this individual is identical to this individual (given linguistically generated relations between that for which we happen to use 'identical', 'individual' and 'this', as opposed to any relations between each of these noises and that for which we use the noises). But the linguistically generated relations that 'Cicero' has to that for which it is used and 'Tully' has to that for which it is used do not make 'Cicero is Tully' self-evidently necessary. For those relations are not LGOs between that for which we use 'Cicero' and that for which we use 'Tully' that make it contradictory for what Cicero prelinguistically is not to be what Tully prelinguistically is. On the other hand, that for which we stipulate the use of 'red' and 'color' (or 'bachelor' and 'male', etc.) *also* happen to be related only by LGOs like more and less precise, vague, etc.; so if 'Something red is something colored' is false, it is self-evident that what 'red' and/or 'color' are used both are and are not what they prelinguistically are.

Since lexicological and grammatical relations are LGOs, the fact that something is an LGO is not sufficient for it to cause self-evidence. 'Cicero is Tully' is not self-evident. The contingent linguistically generated relation of the name, 'Cicero', having the same use as the name, 'Tully', differs from the linguistically generated relation, more-and-less predicative precision, that grounds the necessary sameness of that for which we happen to use 'red' and 'color'. With respect to our knowledge of necessary truths, what holds for lexicological relations holds for grammatical relations as well: Both are LGOs causally dependent on features that a language need not share with other languages able to express the same thing. But a language could not express the same things that 'red' and 'color' express without, *ipso facto*, expressing things with the linguistically generated features that make 'Red is a color' self-evidently necessary. Knowledge that 'Cicero is Tully' depends on knowledge that the contingent language-specific relation of each of these noises to that for which it is used in our language happens to be what it is. But if another language can express what we express by 'red' and 'color', someone could be lexicologically and/or grammatically mistaken about the ways it expresses the color red without being mistaken about the truth we express by 'Red is a color'.

Nor can we need a formula expressing the difference between the ways some LGOs can cause necessary truth and self-evidence and some do not for the necessary truth of 'Red is a color' to be self-evident to us. Such a formula would be part of the after-the-fact causal explanation of the difference between knowing 'Cicero is Tully' and 'Red is a color'. That very explanation must imply that the prior fact being explained, our knowledge of the self-evident, did not occur by means of such a formula, which would amount to knowing by means of a criterion, or by means of knowledge of something other than a truth's own words. And once we know that a sentence is necessarily true, we can know that, on the hypothesis (the truth of which we need not know) that sentences using other lexicological or grammatical structures happen to express the same thing, those sentences are necessarily true also.

Quine thinks that knowing ‘analytic’ truth would require ‘inexplicable insight’ into ‘reality’.[43] But if (I) and (II) are true, it would be inexplicable if we were *not* aware of an LGO that makes ‘Red is not a color’ contradictory, once we understood how ‘red’ and ‘color’ are used. On the assumption that LGOs like those exemplified in (I) sometimes occur, it happens that when we understand that for which certain words are used, we cannot avoid being aware of LGOs that make the truth of certain sentences self-evidently necessary. Our failure to be aware of the LGOs that make some truths self-evident would contradict the hypothesis that we understand that for which those words are used. So simply understanding how we use ‘red’ and ‘color’ must tell us that they are ways of expressing the same feature of experience, ways that differ only in their post-experiential linguistic properties. If we are aware of the way we use these words, we cannot avoid being aware of relations that make ‘Red is a color’ true unless that for which we use ‘red’ and/or ‘color’ is not what it prelinguistically is. But if we needed criteria to know that the relation between the ways we use words is the kind of LGO exemplified in (I), that knowledge would require drawing a conclusion from knowledge of how we use *other* words, which would amount to needing criteria for identification to judge a truth that was knowable merely by understanding how its words are used.

Further, no understanding of usage is needed for knowledge of ‘Red is a color’ other than that needed for knowledge of contingent empirical truths like ‘My toe is red’ and ‘That color is red’. For if (I) is true, no understanding of the uses of ‘red’ and ‘color’ *can* be needed other than that needed to know ‘My toe is red’, ‘That color is red’, etc. Nor do we need a psychological account of how we know the uses of ‘red’ and ‘color’ in order to know that the understanding of usage required for knowing ‘Red is a color’ is the same kind as that required for knowing such contingent truths.

And unless a truth is specifically about self-evidence (or epistemic necessity) or contradiction, recognizing that its opposite is self-evidently contradictory does not depend on knowledge of how words like ‘self-evident’ and ‘contradiction’ are used. We do not even need these words in our vocabulary until after we discover that some sentence can fail to be true if and only if something is and is not what it is. Once these words are in the vocabulary, criteria for their use would tell us the lexicological facts that we can use these noises to express certain things. But we must be able to be aware of those things independently of knowing that they fulfill the criteria for using those words; if we could not be aware of those things independently, we would have nothing to apply those words to.

Likewise, if we had criteria for applying a phrase like 'less precise than', to the relation between these uses, the criteria could only tell us the lexicological fact that we can use the noises 'less precise than' to express what we were already aware of in being aware of the relation between what it is to be something red and what it is to be something colored. That is, to employ the criteria for 'less precise than' we would have to know that the criteria obtain independently of knowing that they are criteria for 'less precise than'. For if 'less precise than' refers to the LGO exemplified in (I), it is a property of that for which we use 'red' relative to that for which we use 'color' such that we could be aware of it only by being aware of that for which we use 'red' and 'color'. So by hypothesis, what 'less precise than' happens to be used for is a relation we have to be acquainted with by being acquainted with other words, before we could employ a criterion for the phrase 'less precise than'. And this relation is what makes 'Red is a color' self-evident. So even if we had criteria for 'self-evident', those criteria could, like criteria for 'less precise than', only tell us how to apply those words to what we already know.

An objection might be that even if we employ a criterion only to apply a word to something known otherwise than by means of the criterion, still the scientific value of criteria for identifying which things are *F*s is well known (though less well known is Goodman's point about the 'gross exaggeration' of the need for criteria of identification[44]). But even if we had criteria for identifying truths whose opposites are self-evidently contradictory, after they are recognized without criteria, the criteria would not serve the purpose that criteria serve in other parts of science. In philosophy, it would rarely be relevant to show that a sentence is self-evident. What we would want to show is that a sentence is necessarily true, that is, that its denial produces a contradiction. We do that by reductio ad absurdum; self-evidence does *not* exempt philosophers from verifying by indirect argument. 'Known by understanding how its words are used' is an after-the-fact psychological concept expressing a causal condition for grasping an argument that denying a statement requires something to be and not be what it is. Only rarely will the concepts of self-evidence, contradiction or their cognates be part of such an argument.

In different epistemic contexts, criteria can range from being needed, to useful but not needed, to not useful, or to not possibly useful. Even when self-evidence is part of an argument, as it is here, criteria for identifying instances of it need not serve the purpose criteria for identification serve in other areas. The argument so far has not required citing instances of self-evidence except for the sake of illustration. We arrived at self-evidence as a causal concept needed to explain something else, knowledge of necessary truth, whose existence was not under dispute *for the purpose at hand* when the concept of self-evidence was introduced. That purpose was to show, against Quine, that *on the hypothesis* that there is knowledge of necessary truth, that knowledge cannot require employment of a criterion. Without such a hypothesis, this discussion would not have taken place; for the discussion was a reply to Quine's critique of such hypotheses.

In sum, if (I) and (II) are true, two things follow without criteria for identification entering the argument: Some sentences express truths that cannot not be true; and we cannot not be capable of knowing that some sentences express necessary truths by the same understanding of usage required for knowing many contingent truths. I unequivocally believe (I) and (II) are true. Since I have used them, however, just as hypotheses (to be dispensed with in Section 14), I have not used (I) and (II) to prove that self-evidence occurs, only that it must occur if certain conditions (which, by the way, can and must occur by causal necessity, if our language is what it is) occur. But why should sentences like (I) and (II) *not* sometimes be true, given the limitations of language? And so why should knowledge of the necessity of truths like 'Red is a color' not sometimes occur, not as a result of a proof, but as a result of the occurrence of the kind of LGOs illustrated in (I)?

11.

But Carroll's Achilles-Tortoise paradox provides a proof that self-evidence must occur if an epistemically *sine-qua-non* case, the grasp of the validity of an inference, ever occurs (as it must whenever human beings make the simplest conscious decisions based on inferences from beliefs already possessed at whatever level of cognition, beyond that of sense perception, language requires). For that paradox shows that an infinite regress occurs unless the grasp of inferential validity is caused solely by an understanding of what is expressed by some of the words in the premises. Again, it is not enough to say that this paradox results from confusing rules of inference with premises. Just as knowledge of a truth by virtue of knowing the usage of its terms cannot consist of knowledge of the usage of a different truth's terms, knowledge of the success of an inference cannot consist of knowledge of the success of a different inference. So unless self-evidence as described above occurs, we could not be aware of the success of any deductive inference. An infinite regress arises if grasping the validity of an inference requires anything more than a grasp of what its premises are and what its conclusion is, as opposed a grasp of whether they are true. So the infinite regress arises unless we know the validity of the rule of inference just by knowing how (at least some of) the words in an inference's premises are used.

The validity of the following inferences must be self-evident, just as is the truth of 'Red is a color':

(A) If *p* then *q*. And *p*. Therefore *q*.

(B) All A is B. And all B is C. Therefore all A is C.Were the validity of the inferential relations in (A) and (B) not known simply by knowing how the words of (A) and (B) are used, we would have to know the validity of inferential relations between the conjoined premises of (A) and (B), respectively, and additional premises stating that the conjoined truth of the original premises requires the truth of their conclusions, additional premises like:

(a) If 'If *p* then *q*, and *p*', then *q*.

(b) If 'All A is B, and all B is C', then all A is C.

And if the latter inferential relations are not self-evidently valid, how do we know *their* validity? 'Self-evident' refers here, not to the premises, but to a relation between the premises and the conclusion. When we learn that premises like (B)'s are true, we must be able to draw the conclusion without separately learning a truth like (b); so we must know the inferential relation just by knowing how at least some of (B)'s words are used.

Our ability to recognize the self-evident is not infallible. If we can have a nonhallucinatory sense experience but not take cognizance of it, we can 'deny' a self-evident truth by ignorance of the question, that is, by thinking we are denying it when we are really denying something else – especially when we read more into a question than is consistent with the truth of the matter. For example, Quine thinks that unless we deny analytic truth, we must admit inexplicable insight. So the ignorance of the question can be very '*learned* ignorance' indeed. In philosophy it usually is; we out smart ourselves.

But the much more common error is to think that something is necessarily true when it is not, since self-evidence can be deceitfully imitated just as genuine sense perception and mathematical proofs can. For example, when knowledge of a truth derives from LGOs, many think it also a necessary truth that the former *truth*, rather than just our *knowledge* of it, is relative to language in ways that contingent truths are not. This is a misplaced causal connection. We understand that in grasping a self-evident truth, or a conclusion from them, we necessarily rely only on our grasp of how certain words are used, but we associate that understood causal connection with the truth known, not just with our knowledge of it. Seeing necessity where there is none occurs in logic and mathematics, but philosophy is more prone to this error since philosophy's words have multiple meanings that are not only closely related but also causally related, and in diverse ways, as *explicanda* and *explicantes*. For example, contributing to the error of making necessary truth linguistic (or conceptual, or a relation of ideas, etc.), is the fact that 'meaning' can refer both to what is intended and to the intending, and in both the lexicological and nonlexicological senses, and in both 'true by' and 'known by "meaning".' So occasions for misplacing causal necessity grow exponentially in philosophy.

12.

Tracing knowledge of logical necessity to our limitations may seem odd. But while God (and angels, according to medieval theology) does not need logic, unless God's knowledge grows by piecing things together, we know truth by piecing together into statements fragments of extensionally and intensionally incomplete grasps of objects in accord with–and so, as a matter of unavoidable fact, under the restraints of–relations and properties that happen to arise, and only arise, from that incompleteness. For just as linguistically generated relations arise unavoidably, they unavoidably restrain the ways we can usefully piece fragments of recognition together. Here 'restrain' means limit the ways we can piece partial grasps of known objects together in statements to achieve a goal for the sake of which we express those partial grasps in language: knowledge of the truth of sentences. Words are tools, and like all tools they have limitations with respect to what you can do with them. You can do this with a tool, but not that. To avoid the limitations of one tool, you can only use another limited tool. But in language changing the tool to avoid its limitations can mean we have only changed the subject, and doing that inadvertently is just a way of being ignorant of the question.

The LGOs in question become objects of cognition as a result of acts performed *for the sake of* articulating prelinguistically known objects. For example, 'red' and 'color' each serve different limited purposes. 'Red' cannot do all the things for us that 'color' can do, and vice versa. A linguistically generated relation pertains to that for which we use 'red' and 'color' as a result of acts performed for the sake of those different purposes. The linguistically generated relation that happens pertain to that for which we use 'red' and 'color' for the sake of their being that for which we use these words is such that there is no difference between that for which we use 'red' and 'color' in their prelinguistic state, since this difference is linguistically generated only.

And we are assuming a person understands how 'red' and 'color' are used; if so, she cannot not know, as long as the latter, secondarily apprehended relations hold, that that for which 'red' and 'a color' are used are not distinct from one another in their prelinguistic state. So she can know that the way that fragmentary objects of cognition are combined in 'Red is a color' is limited to having only one of the values true or false, namely, the value true, because she is aware of the following: The prelinguistic feature we articulate by 'red' is the same, in its prelinguistic state, as a prelinguistic feature we accurately describe by 'a color'. She can know this limited sentential truth because she knows the pre-articulated identity of what is diversely articulated by 'red and 'color' in 'Red is a color'. And by hypothesis what is diversely articulated is limitedly articulated.

Now assume this person understands that we use the word, 'not', for another LGO needed by piecemeal knowers, negation.[45] What is that person supposed to think about the way fragmentary objects of cognition are combined in 'Red is not a color', if she considers this linguistic construct to exist for the sake of achieving the goals of sentential truth and knowledge of sentential truth? Assuming that she is attentive to what she knows about the linguistically generated relation for which we use 'not' and the linguistically generated difference between that for which we use 'red' and 'color', it follows that she knows (or at a minimum *can* know) that the way of combining partially grasped objects in 'Red is not a color' does not achieve one of the goals for the sake of which we use language, sentential truth. For it follows that she knows that a linguistically generated relation between the ways we use 'red' and 'color' happens to place a restraint on the ability of combining that for which we use 'a color' with that for which we use 'not' in 'Red is not a color' to still serve the purpose of letting us know of a pre-articulated identity of what is diversely objectified by 'red' and 'not a color' (or a prelinguistic nonidentity of what is diversely articulated by 'red' and 'a color').

And why shouldn't she be able to know that the way she uses 'not' makes 'Red is not a color' unable to be true unless that for which we use 'red' and/or 'color' is not what it is? Because she would have to have inexplicable insight into the natures of things? On the contrary, it is because her insight into the natures of things is so partial and fragmentary that what she happens to use 'red' and 'color' for have a relation between them that, by being what it happens to be, makes that for which these words are used differ only in a linguistically generated feature and so not in the extralinguistic feature they articulate. And it is because she uses 'red' and 'color' for the limited purposes that cause this LGO to come into apprehension, that she cannot not know, when aware of how she uses those words, that this relation so limits their usefulness that 'red' cannot articulate anything prelinguistically different from what 'a color' also articulates.

And it is a direct result of her knowledge being essentially partial and fragmentary that she needs to be able to use signs for other-than, signs like 'not'; for parts and fragments are, by hypothesis, *other than* one another. Negation just happens to be, though it need not be, the intended effect for which we use 'not'. And the *intended* effect of 'not' is the only one relevant here, since we are talking about that for which we purposefully use a word. Attempts to achieve intended effects can fail. But failure here would amount to no more than being lexicologically mistaken about how we are using 'not'. For we are assuming that someone understands how we use 'red', 'color' and 'not', and asking if on that hypothesis she can fail to know that those uses require 'Red is not a color' to be false. In fact, it would be the *truth* of 'Red is not a color' that would require a failure to achieve an intended effect, since it could only be made true by changing the subject from that which we now intend by our use of 'red', 'a color' and/or 'not'.

So not only do the conditions we are presupposing, knowledge of that for which certain words are used, cause her to know that 'Red is a not color' is false, they cause her to know that if it was true, that for which we use 'red' , 'color' and/or 'not' would be what it is and not be what it is. Therefore, she would not only know that 'Red is a color' is true but that it is true on penalty of contradiction, necessarily true.

Given the extremely natural and innocuous character of the conditions we are presupposing, should there be any mystery why the opposite of 'Red is a color' is contradictory and why she can know that it is contradictory just by knowing how certain words are used? At some level there shouldn't be any mystery. But this is philosophy isn't it? There are reasons why after 2500 years we are still incomparably better at discovering problems than discovering solutions about which sociologically defined 'experts' can achieve long-standing and cumulatively growing consensus, as experts do in science.

One of the most important of these reasons is that our questions are either about or lead us quickly into issues about things that are presupposed by our habitual and all-but-universal methods of answering other questions. Since philosophy must ask questions about the presuppositions of normal methods for answering questions, we can't use those methods to answer philosophical questions. That is also an important reason why we so readily succumb to the temptation to apply new and more sophisticated methods of answering other questions to philosophical questions, since our less sophisticated methods have failed to answer them. So we willfully overlook the fact that, since we cannot create new methods *ex nihilo*, our creation of new methods must rely on presuppositions of the same kind that prevent other methods from being applicable. By 'willful' I don't mean we so err by commission, just by omission. In the enthusiasm due to our justifiable intellectual dazzlement with the power of new methods, and our understandable impatience to be rid at last of chronically intractable problems, we neglect to *choose* to invest as much effort in examining the presuppositions of the new method and of applying it to those problems as we do in applying it to those problems.

Examining rock-bottom presuppositions will always be extremely difficult, for the reason stated and even deeper reasons.[46] Knowledge isn't just assent to truth; it is assent to truth caused by awareness of evidence sufficient to exclude the opposite from truth. In philosophy, finding that evidence almost always requires walking on a razor-thin wire hovering over the opposite sides of a false dichotomy, like the laws of thought/abstract objects dichotomy about the subject of logic, or over the opposites sides of an apparent contradiction. We avoid falling off the wire into either abyss, with no net below or parachute above, only by making distinctions that are agonizing because our habitual, all-but-universal intellectual methods don't require distinctions that are so exacting. And solving a philosophical problem not only requires binoculars strong enough to let us see the razor-thin but all important wire we have to walk on but also requires an exceptionally long attention span while we work out arrangements of words that can do justice to the subtlety of the problem. Since philosophical problems are so often connected, however, after solving one problem we have to make another exceptionally long extension of the *same* attention span to deal with the next problem. Given those preconditions for philosophical success, although we each can have correct opinions about many philosophical questions, the word that does justice to how we each can achieve genuine knowledge about more than a few philosophical questions is a simple one: luck.

In the case of 'Red is a color', again, there should indeed be no mystery why a sufficiently able and attentive user of the language can know that it must be true. But it does not follow that philosophers should be able to explain that absence of mystery without great difficulty; philosophers should find articulating reasons for the absence of mystery at that level of knowledge very difficult at their own level. For example, that for which we use 'a color' is a prelinguistic object. Now combine that object with something that is merely a linguistically generated (or at least cognition-constituted) object, negation. Since the only difference between 'a color' and 'not a color' is a word, 'not', for a merely linguistically generated relation, shouldn't the *only* difference between that for which we use 'a color' and that for which we use 'not a color' be something linguistically generated, something the comes from the side of the language using-agent, as does the difference between that for which we use '6 times 6' and '6 times itself', where that for which we use 'itself' is the LGO, identity? But what we want to say by adding 'not' to 'a color' is precisely the opposite of 'the same as a color prelinguistically, since only distinct from a color by an LGO'. We seem to be getting something out of adding a word for an LGO to 'a color' that is the opposite of what the word can put into it.

Voila, a philosophical problem we didn't see coming as we trustingly explained, by means of the nature of LGOs, why the falsehood of 'Red is not a color' is self-evidently necessary. How difficult would it be to work out the solution to this problem? We could start by pointing out the (causally necessary) truth that the way linguistically generated relations originally come into apprehension requires that at least one of the *relata*, one term of a relation, be something prelinguistic. For LGOs come into apprehension as a result of acts whose purpose is to communicate about the prelinguistic. So appearances (but strong appearances) to the contrary, there is no conflict between the idea that something as fundamental to a limited knower as negation is linguistically generated and the idea that it initially comes about precisely to have its effect on objects that are not just linguistically generated, comes about to do the work of being a relation to a nonlinguistically generated *relatum*. So why shouldn't the addition of 'not' to 'a color' be able to say the opposite of 'the same as color prelinguistically, since only distinct from color by an LGO'?

In the thing/object vocabulary, for object 1 to be distinct from object 2 only by cognition-constituted values is one thing; for the way we objectify 1 to be distinct from the way we objectify 2 only by the use of a word or words for cognition-constituted values in another. Object *A* differs from object *not non-A* only by the cognition-constituted value, double negation. So what is objectified by 'A' is extracognitionally identical with what is diversely objectified by 'not non-A." But given the cognition-constituted value for which we use 'non," namely, negation, what 'A' and 'non-A' objectify are diverse more than just by being terms of these diverse relations of objectification. They are diverse extracognitionally, where 'extracognitionally' means that *A* and *non-A* are distinct more than just by being terms of the diverse cognitional relations, referred-to-by-'A' and referred-to-by-'non-A.'

'Non' happens to be used for the relation of other-than to whatever argument 'non' takes in a particular case. Since language is public, the first argument 'non' takes are some real, cognition-independent existents, and 'non' signifies the relation of other-than to a real existent. If so, 'A is not non-A' tells us something true about real existent, A, in its state of being more than merely conceived or imagined. Subsequently, 'non' can have cognition-constituted objects as its arguments. But 'A is not non-A' is also true of any cognition-constituted object because, if it were not, then either negation, which is what we happen to use 'non' for, both is and is not negation, or real existent A could also be non-A. For that for which we intend to use 'non' does not change just by our going from using 'A' in 'A is not non-A' for real a real existent to using it for a cognition-constituted object. The cognition-constituted relation, negation, is first of all an instrument for objectifying what extracognitional things are; for they are either A or non-A. Since all language derives from language for objectifying real existents, the first instruments by which we objectify cognition-constituted objects are the same as those by which we objectify real existents. That for which we use 'not' and 'non' causes the necessary identity of what are objectified by 'A" and 'not non-A' in the case of real existent, A; so it also causes the necessary identity of what are objectified by 'A" and 'not non-A' in the case of cognition-constituted object, A.

The principle of noncontradiction, whose necessity derives from the cognition-constituted object, negation, is a metaphysical truth about (actual or possible) real existents, not a truth of the science of logic. The reason the principle of non-contradiction must also be true of cognition-constituted objects is that, if it were not, it would not be true of real existents, since words for negation have the same cognition-constituted effect in both cases, the effect of objectifying whatever is other-than the argument negation happens to take.

13.

Is the problem now solved? Well, might not someone ask why, since relations like negation and precision are LGOs, we shouldn't say that that for which we use 'not' and 'precision' differ only by LGOs. And if differing only by LGOs requires that for which we use 'red' and ‘color’ to be extralinguistically identical, why aren’t that for which we use ‘not’ and ‘precision’ extralinguistically identical despite being nothing but LGOs? So here is another problem we might not have seen coming as we innocently but with much effort forged arrangements of words to offer a reason why ‘Red is not a color’ is self-evidently necessary. Must we now replace those arrangements of words with different ones, or abandon the quest? No, but we must make the even more back-breaking effort to put words together in ways that (1) save the work already done, (2) state why that work does not lead to this contradiction, and (3) do not lead to further contradictions (though this being philosophy, they will almost certainly lead to further strong *appearances* of contradictions — philosophers will never be out of the paradox explaining business).

And that new effort will require a further extension of the same attention span required to work out the answer to the prior problem about getting more out of adding the LGO, negation, to that for which we use ‘a color’ than we put into it. For the solution to that problem, that LGOs initially occur such that on at least one of their terms is prelinguistic, is the basis of the solution to our new one.

Wittgenstein’s private language argument tells us more than that language initially occurs for the sake of communicating about public objects; it tells us that language occurs to communicate about objects that are public in the sense of existing extracognitionally, as opposed to being merely imagined or conceived.[47] In articulating prelinguistic objects of cognition, we give ourselves new objects, LGOs, that don’t exist extracognitionally. We can then extend language to LGOs and use words for them. But since language initially exists *for the sake of* communicating about real, extracognitionally existing objects, the language we initially use for communicating about LGOs must be derived from, and so based on, language for real existents. In the process of extending language beyond its initial purpose, we invent new linguistic devices, as in mathematics and systems for verifying computationally in logic, and so new LGOs, for example, new kinds of sets, including a set with no members and sets whose members are sets, beyond the kinds constituted by the extensions of our initial predicates. But at the *epistemic* level of how we know that a step in a computational proof is correct according to the rules, we can never completely get away from the LGOs that occur for the sake of articulating real existents, since their articulation is causally fundamental to language.

We might imagine that we can kick the ladder of the prior LGOs away when we have constructed new devices to replace them, and in some sense other than *epistemic* we might be able to kick away that ladder. But at each step in the construction of a new device we have to be able to know the truth of judgments about the process, and those judgments must be made using the previous devices, if only because the new devices are not yet established. And in order to know the sense in which it is legitimate to kick the latter away, we must know the truth of judgments made using previous devices, if only because the assumption must be that we don't replace them with new devices until we already know that we can safely kick the ladder away. But more fundamentally, whatever we make think of some philosophers' practice of analyzing ordinary language in terms of 'rules', the construction of new devices will involve rules, by hypothesis. And rather than rules applying themselves, we now know that recognizing that we are applying them correctly, as we must in order to use the new devices after they are created, requires inferences whose validity we grasp by understanding that for which we use words possessed prior to the new rules.

Hence, to come back to our problem about LGOs like negation and precision, when we extend language for real existents to LGOs we should not be surprised to find that we must still use LGOs like negation and precision in speaking of LGOs themselves. So we must seek further painfully crafted arrangements of words that do not imply that objects like negation and precision, which can only differ by LGOs because they are only LGOs, must be identical, since only differing by an LGO is what makes red and a color identical.

How about: Being prelinguistically distinct, red and green differ from one another by more than a mere LGO. But their prelinguistic difference does not imply that they differ from that for which we use 'color' by more than an LGO. So red and green do not differ solely by an LGO attributable to them as a result of the way they are articulated by 'red' and 'green'. But as a result of the way they happen to be articulated by 'color', they do differ solely by an LGO from that for which we use 'color'. Negation and precision are linguistically generated relations attributable to objects other than themselves. As such, they are different LGOs, different ways objects can be related. And that difference is *presupposed* by the way they are articulated by 'negation' and 'precision', respectively; for that these different relations happen to be what they are is what we intend to communicate by 'negation' and 'precision'. So that difference does not pertain to them *only as a result* of the way they are articulated by 'negation' and 'precision'. But that presupposed difference does not imply that they differ from that for which we use 'LGO' by more than a linguistically generated relation, namely, being-less-precise-than, pertaining to them as a result of the way we use 'LGO'. Like red and green in relation to that for which we use 'color', negation and precision do differ solely by the linguistically generated relation of being-less-precise-than from that for which we use 'LGO'.

What it means to say that red and a color must be identical is that that for which we use 'red' and 'color' differ solely by LGOs pertaining to them only as a result of being articulated *by certain words* (in this case by 'red' and 'color', and possibly by other equivalent linguistic devices), because those words happen to be used in *certain ways*. But red and green do not differ solely by LGOs pertaining to them only as a result of being articulated by 'red' and 'green'. They are non-identical colors despite being identical with respect to that for which we use 'color'; that's why 'color' is less precise than 'red' or 'green'. Likewise, the LGOs differentiating that for which we use 'negation' and 'precision' do not differ only by LGOs negation and precision acquire by being articulated by 'negation' and 'precision'. So far, then, the LGO analysis does not force us to consider negation and precision 'prelinguistically' identical where 'prelinguistic' means 'outside of being articulated by specific devices like the noises "negation" and "precision"'.

But the LGO analysis still explains identity when it is necessary, as in the cases of

'Negation is an LGO' and 'Precision is an LGO'. 'Red is a color' and 'Green is a color' are both necessarily true, but the fact that red and green each differ only by an LGO from that for which we use 'a color' does not make red the same as green, other than with respect to the limited accuracy of 'a color' and hence with respect to the limited truths: 'Red is a color' and 'Green is a color'. Likewise, negation and precision each differ only by the linguistically generated relation of more-and-less-precision from that for which we use 'LGO'. So each is identical with that for which we use 'LGO' other than with respect to a linguistically generated relation that they (naturally and by causal necessity) acquire by the fact that we so use 'LGO'. But the fact that each of negation and precision differ only by a linguistically generated relation from that for which we use 'LGO' does not make them the same other than with respect to the restrained accuracy of 'an LGO' and hence with respect to the limited truths: 'Negation is an LGO' and 'Precision is an LGO'.

14.

The limitations we have been talking about are entirely *natural.* Because they are also inescapable, we call them or their effects 'necessities'. Necessity that is an effect of LGOs has been called 'logical' necessity. We could even call the necessity of 'Red is a color' logical, even though the statement is about what the prelinguistic objects red and color are. But let us now restrict 'logically' necessary to statements whose truth is not just caused by LGOs, as in 'Red is a color', but statements whose necessary truth is due to their mentioning, their use of words for, the necessity-causing LGOs that come from our limitations as knowers.

To see what the LGO analysis has to do with necessity that is logical in this sense, consider the following:

(1) A is not non-A

Contrary to 'Red is a color', (1) uses words for something that is an LGO, negation. So (1) mentions an LGO, while 'Red is a color' does not. But due to the ways (1) mentions an LGO, any difference between the ways we use 'A' and 'not non-A' is like that between the ways we use 'red' and 'color'; it is on the side of our manner of expressing something, not on the side of what the expressed is apart from being so expressed. So while 'not' and 'non-' play their current roles in our tools for expressing things, something that we express by 'A' can differ from what we express by 'not non-A' only if something is not what it is, that is, only if A is not A or negation is not negation.

To know the necessary truth of this sentence, we have to know the contingent grammatical fact that the negation expressed by 'non-' is within the scope of the negation expressed by 'not'. That is, we have to know the grammatical fact that the repetition of signs for negation does not here have a use like that which the repetition of negatives in 'No, no!' usually has. But we could be wrong about these grammatical facts without being wrong about any necessary truth. And if a phrase like 'No, no' was being used to put the second negative in the scope of the first, we could have behavioral evidence of that, just as we have behavioral evidence that people use ungrammatical forms like 'There isn't no . . .' to add emphasis to the first negative, rather than cancel its effect by the second. Likewise, if we were lexicologically mistaken and thought 'and' was used the way we use 'or,' we could think 'A and not A' was necessarily true rather than necessarily false. But there could be behavioral evidence that the mistake was about a lexicological fact, and so not about a logically necessary truth. (So this defense of our knowledge of necessary truth is not psychologistic and does not depend on a psychology of cognition created for the purpose.)

Our grasp of that for which we use 'not' and 'non-' can reveal the necessary truth of (1) since we cannot have that grasp without recognizing that the meaning of 'A' differs from that of 'not non-A' only by the latter's repeated mentioning of an LGO in such a way that each mention cancels the effect of the other. So we cannot understand how we use 'not' and 'non-' without understanding that 'A' and 'not non-A' do not differ in that which is expressed but only in the way in which it is expressed. Some LGOs may be too complex for a mere recognition of how a word for an LGO is used to reveal each necessary truth into which the LGO enters. But there can hardly be a simpler linguistically generated object than negation.[48] So it would be inexplicable if we were not capable of knowing the necessary truth of 'A is not non-A'. And to know that 'A is not non-A' is necessarily true, we do not and cannot require a criterion by which to judge that 'not' and 'non-' are used for an LGO (nor to recognize them as Quine's 'logical' particles).

Unlike in 'Red is a color', where the necessity-causing LGOs are transparent, in logically necessary truths the LGOs are visible because mentioned. *That is why hypotheses (I) and (II) are dispensable for analyzing logically necessary truth*. But they were important for showing that LGOs and self-evidence are not ad hoc devices for defending logical necessity. LGOs are natural and ubiquitous, whether mentioned or not. Furthermore, if awareness of LGOs can cause awareness of necessity when the LGOs are not even mentioned, as it can if (I) and (II) are true, it certainly can when they are mentioned.

Again, the multiplicity of kinds of negation is not a problem. We addressed that question in Section 1 concerning ECQ. In addition, it is worth mentioning that Routley and his colleagues adequately showed, using W. E. Johnson's terminology without all its baggage, that if 'the scope or range (as between predicates and sentences) of the negation is simply left open,' we are dealing with a syntactical or semantical *determinable*: normal negation (though unnecessary paraconsistency due to the ECQ problem forced Routley to falsely distinguish normal from 'classical').[49] It can be made *determinate* by specifying its scope, perhaps in conjunction with a modifier, as by using 'not possibly' for intuitionistic negation. The ways signs for determinate negations are used include determinable negation, just as the way 'red' is used includes the way 'color' is used. And whoever can know the kind of contingent information (for example, that something is not A) communicated by signs for normal negation can know the necessary truth of 'A is not non-A'.

Quine asked, in effect, how can we predict that something science accurately express by 'A' in the future will not also be something accurately expresses, at the same time, by 'non-A'?[50] We can predict it by knowing what 'not' and 'non-' happen to express *now*. As long as negation signs have the use they now happen to have, to learn that what A is at any given time is also not what A is at that time, which achieves no epistemic goal. As any relation is a relation-*to* something and functions are functions-*of* arguments, negation is a negation-*of* what something is, whether that something be a prelinguistic object, an LGO, or anything else expressible in language. 'Non-A' says 'other than what A is'. If our best future theory would say about some A that what A is is not what A is, that would show our inability to express knowingly what A is, not just our limited manner of knowing and expressing it, which is true of all our knowledge of what anything is.

Still, due to our inability to know completely what anything is, science could find itself forced to use a concept concatenating objects that are (at least) implicitly contradictory. In fact, it has already done so, in finding uses for the implicitly contradictory concept of the square root of negative one. Section 1 of this study has shown the reason this is possible: it is *not* the case that anything whatsoever follows from any contradiction.[51] And not just any contradictory combination of notes need prove useful in science or any other endeavor. Although we have found the combination of negative-one and number-that-has-a-square-root (number-resulting-from-the self-multiplication-of-another-number) helpful, we have not found the combination of square and circle helpful other than as an example of the useless. So should future science find it helpful to use a contradictory combination of notes, we as the entities who consciously achieve science would not be forced to consider that combination to have any more metaphysical weight than do imaginary numbers.

Quine also thought that putting logic on a continuum with the rest of science required rejecting necessary truth and denying that logic's truths differ from science's by being 'linguistic' in some sense unique to them. Due to Kripke it is now respectable to contemplate necessary truths being on a continuum with other scientific truths. Here I have argued, in addition, that logically necessary truths can be (a) epistemically necessary, to use Kripke's words, and yet (b) on a continuum with the rest of science both with respect to being, *pace* Quine, (c) no more linguistic than other truths and to being (d) fallibly known. But not all truths that are logically necessary, in the sense of truths whose necessity is caused by the way they mention LGOs, are truths called logical because they belong to the science usually called 'logic,' the study of valid inference. Let us henceforth restrict the word, 'logic,' and its cognates to that sense.

The necessity of

(2) Nothing both is and is not at the same time in the same respect.

derives from the nature of the linguistically generated (or cognition-constituted) relation, negation; so does our knowledge of its necessity. But since language is public, the relata on one side of our primary linguistically generated relations must be things cognized as real existents, that is, as having a status that is more than just being conceived or imagined. So (2) does not inform us about negation and what negation does to the relations between the truth values of the premises and conclusions of inferences. It informs us about a condition excluded from possibility for any more-than-just-cognitional reality, the condition of both existing and not existing in the same way at the same time. The statement informs us of something necessary for any existent insofar as it is an existent. Although its necessity derives from its mentioned LGOs, it is a metaphysical statement about beings as pre-cognitional, not about beings as cognized in valid inferences. It remains to be seen how logic's principles of valid inference can be self-evidently necessary.

Nor is (2) a principle of *inference* as far as the epistemic type traditionally known as logic is concerned; by that standard, calling it, or '~(p~p)', an inference principle is an anomaly (see above, Section 7). It remains to be seen how logic's principles of valid inference can be self-evidently necessary.

15.

Carroll's Achilles-Tortoise paradox showed us that the validity of inferences like the following must be just as self-evident as are the truths of 'Red is a color' and 'A is not non-A':

(A) If *p* then *q*. And *p*. Therefore *q*.

(B) All A is B. And all B is C. Therefore all A is C.

We can see how the validity of these arguments is knowable simply by knowing how their words are used by formulating the inference principles we implicitly know in knowing that the conclusions of arguments like (A) and (B), respectively, are true if their premises are.

(a) If 'If *p* then *q*, and *p*', then *q*.

(b) If 'All A is B, and all B is C', then all A is C.

In (a) and (b) we have single sentences, not relations among three sentences. The truth of (a) and (b) derives from their use of words for relations that are LGOs ('if . . . then', 'is', 'and', 'all'), as does the truth of 'A is not non-A'. And these LGOs make sentences (a) and (b) *necessarily* true; for unless they are true, that for which we use one or more of their words is and is not what it is. And since the relations that make these truths necessary are that for which we use words like 'all', 'is', 'if . . . then', etc., we cannot not be aware of those relations in knowing how the words of these sentences are used. So in knowing how those words are used, we cannot not be aware of LGOs that, as a matter of fact, make it possible for those sentences not to be true if and only if one or more of those LGOs is and is not what it is. The same LGOs that make (a) and (b) self-evidently necessary truths make inferences (A) and (B) self-evidently valid by making self-evident the necessity of (A)'s and (B)'s conclusions being true if their premises are. We know that arguments (A) and (B) are valid by knowing that if the truth of their minor premises ('p' and 'All B is C') and the falsehood of their conclusions ('q' and 'All A is C') would contradict what we assumed in their major premises ('If p, then q' and 'All A is B'). For by knowing how their words are used, we know that if (A) and (B) were not valid inferences, one or more LGO would be and not be what it is.

When we believe that an inference is valid, we are believing in the truth of the inference principle implicit in the inference. But since the sole evidence for the truth of an inference principle is what words are used for, the evidence can be of only one kind: evidence that the opposite would make what a word is used for not be what it is. So belief in the validity of an inference is belief that the opposite of the inference principle is or implies a contradiction. And *at the time* when the cause of that belief is the recognition that relations between meanings allow the opposite to be true only if at least one of those meanings is not what it is, the state we are calling 'belief' should be called *knowledge*, that is, belief caused by awareness of evidence sufficient to exclude the opposite from truth. We can *later* justifiably *believe* that the earlier state was knowledge. The later belief may be false; for self-evidence can be imitated. But our ability to revise the later belief does not show that we did not have knowledge earlier; for we can also err in revising such a later belief.

Note that the way LGOs cause self-evident necessity in (A) and (B) parallels the way they were hypothesized to cause it in the analysis of 'red' and 'colored'. How we use 'colored' and how we use 'red' differ only by a linguistically generated relation such that 'X is colored' must be true if 'X is red' is, but not vice versa. In (A) and (B) the truth-values of the conclusions and the conjoined truth-values of the premises are distinguished solely by their being the truth-values of linguistically generated structures employing the LGOs for which we happen to use 'if . . . then', 'and', 'all' and 'some' in such a way that the conclusions must be true if the premises are conjointly true, but not vice versa. The *truth* of predicating 'colored' of something does not depend on red's being one of the things color is distinct from only by an LGO; something can be colored and not be red. Likewise, the truth of (A) and (B)'s conclusions does not depend on their having the linguistically generated status of being that-which-is-validly-inferred-from-the-premises-in-(A)-and-(B).

We *know* that 'Red is a color' is true because the LGO distinguishing redness and color is such that in knowing what 'red' says we know everything 'a color' says, though 'red' says more. The LGOs distinguishing the premises from the conclusions in (A) and (B) are such that joint knowledge of the premises causes knowledge of the conclusions, but when we have joint knowledge of the premises we are aware of more than we are when we have knowledge of the conclusions alone. Whoever knows the joint truth of 'If *p*, then q' and 'p' cannot not know both as much as and also more than anyone who knows only the truth of 'q'. Whoever knows the joint truth of predicating 'B' of every 'A' and 'C' of every 'B' cannot

not know both as much as and also more than anyone who knows only the truth of predicating 'C' of every 'A'. (We can know how 'red' is used without knowing how 'color' is, and vice versa, and so not know that 'Red is a color' is true. But the necessity-causing LGOs in (A) and (B) are mentioned, as negation is in 'A is not non-A'. So if we know how the premises' words are used, we cannot not know that what the conclusions say is true if what the premises say is true.)

Since we can err, however, about something's being necessary, we rely on computational methods of verification wherever we can. Those methods do not eliminate the need for knowledge of necessity, and so for self-evidence, since recognizing that a step is an instance of a rule presupposes an implicit knowledge of a necessarily true inference principle. The clarity and rigor of computational procedures causally presupposes implicit knowledge of self-evidently necessary inference principles. Still, computational methods greatly reduce the chances for error by minimizing self-evidence about more abstract and foundational matters in favor of the self-evident validity of inferences about formulas being instances of general rules about empirically identifiable arrangements of marks. (To the extent that they use uninterpreted symbols as placeholders, the formulas of computational logic are 'abstract' in at least one of the nondenumerably infinite meanings of that word. But what allows us to verify, in a way that minimizes error regarding the self-evidently necessary, that a step applies a rule correctly is the empirically *concrete* character of the arrangements of marks the rules concern.)

16.

After all of the above, should we say that logical knowledge, in the sense of the knowledge of valid inference that is prior to and presupposed by knowledge of computational correctness, is about laws of thought or about abstract objects? The answer for both sides of the dichotomy is yes and no (as the answer to most philosophical questions must be).

The necessary truths of logic are not laws of 'thought'. They are laws of that about which we think (use language): first, about the states of affairs, events, entities and their features, of public prelinguistic experience; then about other cognized objects, like LGOs, imaginary objects, etc. But unlike, say, the laws of physics or mathematics, logic's laws express things that pertain to objects about which we use language only as a result of the fact that they are objects about which we think with the help of language only as a result of the fact that they are objects about which we think with the help of language. Those laws concern, not our thought, but relations that hold between objects we think about only as a result of the fact that they are objects we think about and of the ways we think about them with the help of language. So the fact that that the laws of logic are causally connected to aspects of our thought has led the 'laws of thought' view of logic to mistake the place both of laws and of thought in the analysis of what the goal of logical knowledge is.

Nor do the laws of logic concern 'abstract entities', be they models, sets, structures, relations of some 'formal' kind, relations in general, or whatever. They concern relations between pieces of information, abstract or concrete information, conveyed in language. For our purposes, there are two ways to look at the information conveyed by, say, 'The toe is red'. That sentence communicates a situation in the prelinguistic world, the toe's being red, as situation that could exist even if language did not exist. But epistemically we can look at the toe's being red as something communicated in language, and as such it has characteristics that it could not have if there was no language. In particular, as communicated by the sentence 'The toe is red', the toe's being red also has the linguistically generated characteristic of being either true or false; it is a bearer of one or the other truth-value. That information is not a bearer of truth-value as a prelinguistic state of affairs; nor is it a bearer of truth-value as that which is communicated by the phrase 'the toe's being red'. But it is a bearer of truth-value as that which is communicated by 'The toe is red'. (Which is simply to say that a state of affairs that is one and the same prelinguistically can be communicated by language in diverse ways with diverse linguistically generated characteristics. And recall that instead of speaking of language and the linguistic, the arguments of the analysis could have used a vocabulary of cognition-constituted objects and cognition-independent or extra-objective existents, where the cognition in question is sufficiently beyond mere sensory cognition to support language.)

There are truths concerning relations between the information conveyed in language, not insofar as that what is conveyed may be a prelinguistic state of affairs, but insofar as a prelinguistic state of affairs is conveyed in language. More specifically, there are truths concerning relations between what is conveyed in language insofar as what is conveyed has the linguistically generated characteristic of being able to be true or false. Among the truths pertaining to information insofar as it is a bearer of truth-value there are necessary truths. And among necessary truths about relations between pieces of language-conveyed information that are bearers of truth-value are necessary truths about valid inference, truths that we traditionally call laws of 'logic'.

For example, one way of combining fragmentary articulations of objects into statements is by use of the linguistic tool, 'All . . . is . . .' . The limited purpose of that tool is to communicate or hypothesize a particular linguistically generated relation between the partial pieces of information expressed by the included tools, the subject and predicate. The fragmentary objects A, B, and C can be combined according to the limited linguistically generated purpose for combining fragmentary objects that is expressed by 'All . . . is . . .', as in (I) 'All A is B' and (ii) 'All B is C' and (iii) 'All A is C'. Because of what that limiting LGO happens to be, when fragmentary objects are hypothesized to be combined in this way, there happens to be at least one limit on the relations between the possible truth-values of (I), (ii) and (iii): On the hypothesis that the first two ways of combining fragmentary objects have the value, truth, the third way can is limited to having the value, truth. If the third way is not limited to the linguistically generated value, truth, the consciously intended limited hypothesis is contradicted. Likewise, one way of combining two fragmentary bearers of truth-value into another is for the limited linguistically generated purpose expressed by the tool, 'if . . . then'. Because of what that LGO happens to be, the use of 'if . . . then' hypothesizes a restricting relation between the possible truth-values of the antecedent and consequent such that, on the hypothesis that the antecedent is true, the consequent is restricted to being true. If the consequent is not limited to the linguistically generated value, truth, the consciously intended limited hypothesis is contradicted.

So there happens to occur explicit human knowledge of at least some of the necessary truths that concern information insofar as it is a bearer of truth-value. We happen to call at least some of that knowledge the science of 'logic'. That knowledge does not tell us whether 'The toe is red' is true or is false. But it is concerned about (a) the information conveyed by that sentence insofar as it is the kind of cognitional object that has the characteristic of being either true or false. Even more specifically, that knowledge is concerned with (b) *relations* between the information conveyed by different sentences insofar as the cognitional objects they convey are bearers of truth-value. Although that science is not concerned with, for example, whether 'The toe is red' is true, it is concerned with the relation between that bearer of truth-value and others, like 'Whatever is red is a capitalist', and 'If the toe is red, it is a capitalist.' And even more specifically, it investigates (c) relations of a certain kind between such pieces of linguistically conveyed information: relations whose terms, their relata, include the possible truth-values of those pieces of information. For example, assuming the truth of 'Whatever is red is a capitalist' and 'The toe is red', that science asks about any relation that may obtain between their truth-values and the possible truth-value of 'The toe is a capitalist'. And it finds the necessary truth that, if the information conveyed by the first two is true, the information conveyed by the third is limited to being true. And that science asks about any relation between the assumed truths of 'If the toe is red, it is a capitalist' and 'It is not a capitalist' and the possible truth-value of 'The toe is red.' And the science finds the necessary truth that, if the information conveyed by the first two is true, the information conveyed by the third is limited to being false.[52]

In other words, we happen to call 'logic' a science that deals with laws of *valid inference*, necessary truths that say if such and such information is true or is false, then this other piece of information is true or is false, respectively. But logic's laws concern relations that hold between pieces of information conveyed in language because of some of the LGOs that pertain to that information, not because of the prelinguistic states of affairs the language in question may communicate. The logician is not concerned with the information communicated by 'The toe is red' insofar as the toe's being red is or is not actually a state of affairs in the prelinguistic world. But as a bearer of truth-value, a sentence's information is the bearer of a linguistically generated relation whose possible values, truth and falsity, are affected (restricted, limited, conditioned) by other LGOs, like negation, conjunction, alternation and implication, or the lack thereof, in the sentence. So its relations to the possible truth-values of other sentences are affected by such LGOs and those of other sentences. The logician is concerned with information insofar as its possible truth-values are so conditioned as a result of LGOs that those values have specific relations to the way the possible truth-values of other pieces of information are conditioned as a result of LGOs (that is, conditioned as a result of limitations that the fragmentation of our articulations of objects imposes on the relations between the possible truth-values of pieces of information).

And since the necessity of logic's truths come from relations pertaining to information as linguistically expressed, not pertaining to it insofar as it has a prelinguistic status, in addition to studying relations between possible truth-values, like negation, conjunction, alternation and implication, logic studies linguistically generated relations, like predicative negation, extension, distribution, supposition, and, perhaps, 'reference' (if we philosophers can ever learn to use that word intelligibly), insofar as they affect, or are affected by, relations of valid inference; for relations of valid inference hold between the possible truth-values of pieces of information because of other linguistically generated properties of information, not because of what information consists of as something prelinguistic.

Is logic, then, concerned with 'abstract objects'? Yes and no. Yes, the relations logic is concerned with are relations between pieces of information conveyed in language considered 'abstractly' in the sense of leaving out the prelinguistic content of that information. But no, logic is not concerned with just any abstract objects and any kind of abstraction. The 'abstract objects' view of logic fails to identify the specific kind of related objects, information as a bearer of truth-value, and the specific kind of abstraction, the leaving out of prelinguistic content. That way of abstractly considering information, however, makes 'abstraction' in another sense helpful in logic, the use otherwise meaningless symbols to indicate that content is left out. And that second kind of abstraction makes possible the use of computational methods as a tool of logic.

Note that this analysis has not depended on having criteria, sufficient and/or necessary, for the use of the words 'logic' and 'logical'. The analysis pointed to the fact of certain conditions, like fragmentation, naturally affecting human knowledge and drew some causally necessary consequences from them, like the occurrence in apprehension of linguistically generated relations that pertain to the objects of knowledge, and pertain first to prelinguistic objects of knowledge. It then drew further consequences like the occurrence in apprehension of statements whose opposites are knowably contradictory because of the nature of certain linguistically generated relations between the possible truth-values of pieces information and the possible truth-values of other pieces of information.

Having argued for the occurrence of truths whose opposites are knowably contradictory for the reason described, I pointed to the empirical truth that we happen to call knowledge of those truths the science of logic. So someone might try to turn this analysis into a provision of necessary and sufficient criteria for the use of 'logic' and its cognates. But that would not be relevant for my purposes. The importance of the necessary connections this causal analysis has argued for is not as criteria for the use of a word but as unavoidable consequences of certain natural conditions of limited human knowledge. Instead of making an empirical lexicological claim about how we use 'logic', I could have made arbitrary lexicological stipulations for neologisms, perhaps 'cigol' and 'cigolal'. Then I would have the right to claim (and a critic the right to deny, of course) that what I have so far shown concerning cigolal knowledge answers those specific questions raised earlier, using the words 'logic' and 'logical', about how certain kinds of knowledge are possible. Or I could have postponed raising those earlier questions until after I had shown, using the 'cigol' vocabulary, what follows from certain conditions of human knowledge. Then I would have introduced those questions about 'logical' knowledge and shown how the work already done on 'cigolal' knowledge answers them.

If there are other questions philosophers want to worry about concerning how to characterize what is or is not 'logical' for the purpose of understanding some aspects of our knowledge, they are welcome to do so. I am not interested in necessary and sufficient defining conditions for the 'logical' but in some specific questions about human knowledge of things we happen to call 'logical', in particular certain questions concerning the epistemic causal conditions presupposed by the kind of knowledge we happen to call 'modern logic'. To answer those question I have sought knowably necessary causal truths, sometimes necessary truths about a cause being or not being sufficient for a particular epistemic effect, sometimes necessary truths about a cause needed or not being needed for a particular epistemic effect.

17.

Finally, this more technical section shows how to extend the LGO analysis to strings of marks defined by one of the most fundamental of computational methods, truth-tables, or better, tables assigning wffs uninterpreted symbols like '1' and '0'. (Actually, they are not entirely uninterpreted. To understand and use the binary 1/0 tables, we have to assume that assigning 1 or 0 to a formula excludes assigning the other. This is another presupposition of using computational methods that is so fundamental that philosophers do not advert to it; and to the extent that we do not, we continue to philosophize superficially.)

An entry in a table for a binary operator on *p* and *q* contains a set of 1/0 assignments to *p* and *q*, respectively. Each set associates one of 1 or 0 with *p* and *q*. For instance, an entry may assign *p* the mark '1' and *q* the mark '0'. The entry will also contain an assignment of either 1 or 0 to a binary operator, so that each set of 1/0 assignments to the atomic formulas is a set which associates one of 1 or 0 with the binary formula. For example,

p	*q*	*p -> q*
1	0	0

Hence, that for which a binary operator like '->' is used is a 4-member set (specifically, a disjunction) each of whose members is a set of 1/0 assignments to the component formulas, *p* and *q*, and to the composed formula using the operator, *p* -> *q*, such that certain component 1/0 assignments assign the formula using the operator 1 and the remaining ones assign it 0. For example,

p	*q*	*p -> q*
1	1	1
1	0	0
0	1	1
0	0	1

These sets, and sets of sets, of 1/0 assignments are LGOs (whether or not all sets are LGOs). 1/0-tables are LGOs since the purpose of the marks making the tables up is not to decorate blank spaces but to communicate something about arrangements of marks having certain forms (forms like *p* -> *q*, ~*p,* etc.).

Given the disjunctions of sets of 1/0 assignments that constitute that for which we use the binary operators, certain formulas using those operators must be assigned 1 and others must be assigned 0, because they could fail to have that status only if those disjunctions of sets are not what they are, that is, only on pain of contradiction. That for which we use *p* -> *q* is a 4-member set of sets of 1/0 assignments to its component formulas such that *p* -> *q* is assigned 1 by any member of the disjunction: *p* is 1 and *q* is 1, *p* is 0 and *q* is 0, or *p* is 0 and *q* is 1; and *p* -> *q* is assigned 0 by the remaining set of 1/0 assignments to *p* and *q*: *p* is 1 and *q* is 0. Therefore, as the way we use 'red' differs from the way we use 'color' only by a linguistically generated relation, so the difference between what is expressed '*p* is 1 and *q* is 1' and by 'a set of component 1/0 assignments that assigns 1 to *p* -> *q*' consists only in the linguistically generated device of the 1/0-table for which we use '->'. That is, the only difference between what is expressed by '*p* and *q* are both assigned 1' and '*p* -> *q* is assigned 1' consists in the linguistically generated relation of the assignment of 1 to *p* and *q* being a member of disjunction of sets that assign 1 to (*p* -> *q*), as the only difference between what is expressed by 'red' and 'color' consists in the linguistically generated relation of being more or less precise than.

A problem with applying the 'differs only by an LGO' account of necessity in this way: That for which we use 'color' differs only by the LGO of less specificity from that for which we use 'red', but that for which we use '*p* and *q* are 1' need not differ only by an LGO like less specificity from that for which we use ' *p* -> *q* is 1'. Unlike that for which we use 'red' and 'a color', each of which expresses what one and the same prelinguistic feature is, that for which we use '*p* and *q* each assigned 1' and that for which we use '*p* -> *q* assigned 1' are different LGOs. In this respect, that for which we use the latter constructions are like that for which we use 'red' and 'green', distinct in their own territories: the latter territory being the domain of the prelinguistic, the former the domain of the linguistically generated. And that for which we use the former constructions are distinct LGOs that need not be related as more or less specific. Unlike what is expressed by 'color' relative to what is expressed by 'red' and 'green', the LGO expressed by ' *p* -> *q* assigned 1' need not be distinct from the LGOs expressed by ' *p* and *q* assigned 1, or *p* assigned 0 and *q* 1, or both *p* and *q* assigned 0' only by *another* LGO distinct from any of these, namely, less specificity between what is expressed by ' *p* -> *q* assigned 1' and by the other expressions. For the LGO we express by '*p* and q each assigned 1' need not be the same as an LGO expressed by "one of the 1/0 assignments that are members of the disjunction of component 1/0 assignments that is the LGO for which use '->'." And if they need not be the same, has this analysis provided conditions adequate to require them to be the same, when those conditions hold?

That for which we use '*p* and *q* each assigned 1' cannot not be *potentially* a member of an infinity of sets of sets of 1/0 assignments to wffs that are *potentially* component formulas of other *potential* formulas using *potential* operators. But until we actually give '->' its normal definition by component 1/0 assignments, there is no basis for asserting that the LGO we express by '*p* and q each assigned 1' is the same as an LGO expressed by "one of the sets of 1/0 assignments that are members of the set of sets 1/0 assignments for which use '->'." On the other hand, when that for which we use 'red' exists, that for which we use 'a color' does not potentially exist; it cannot not *actually* exist. For that for which we use a 'color' is not, like that for which we use '*p* -> *q* assigned 1', an LGO; rather, since that for which we use 'a color' differs from an actual feature only by the LGO of less specificity in the way the feature is articulated, that for which we use 'a color' must be prelinguistically identical with what that actual feature is.

But when we give '->' its usual 1/0-table definition, the component 1/0 assignments and the relevant set of sets of those assignments are, ipso facto, no longer merely potentially apprehended LGOs. (Whether sets 'exist' when not apprehended is not relevant to our epistemic question about LGOs: When is there an actually apprehended object of cognition, or at least an object, like extension, with a capability of being apprehended, that results from actually occurring uses of language?) For unlike 'Red is a color', where the necessity-causing LGOs are not mentioned by their own words, '*p* and *q* each assigned 1' and '*p* -> *q* assigned 1' do nothing but mention LGOs explicitly. In this respect the way we know the necessity of assigning 1 to what '((*p* -> *q*)*p*) -> *q*' expresses in computational language is like the way we know the precomputational necessary truth expressed by 'If wet then slippery, and wet; so slippery'. In both cases, the necessity causing LGOs are explicitly mentioned.

And on the hypothesis that someone understands that for which '->' happens to be used, she cannot at the same time not know both that (I) assigning 1 to *p* and *q* assigns 1 to *p* -> *q* and that (ii) what is expressed by '*p* and *q* each assigned 1' differs from what happens to be expressed by 'a member of the set of component 1/0 assignments that assigns 1 to ->' only by the very LGO for which the latter expression is used, and not just by an unmentioned LGO associated with that for which a word is used as in the case of 'color'. So given the LGO for which we ordinarily happen to, though we need not, use '->', she knows that assigning 1 to *p* and *q* could fail to be a member of the set of component 1/0 assignments that assign 1 to *p* -> *q* only if the disjunction of component 1/0 assignments that assigns 1 to *p* -> *q* is and is not what it is. And she knows that assigning 1 to *p* -> *q* differs from assigning 1 to each of *p* and *q* only by an LGO such that what is communicated by assigning 1 to *p* -> *q* is communicated by assigning 1 to each of *p* and *q*, but the latter communicates more than does the former. For in knowing that *p* -> *q* is assigned 1 she knows that at least one member of a disjunction is also assigned 1, but she does not know which member of the disjunction.

To show how to extend this kind of analysis to more complex formulas, I will examine modus ponens, ((*p* -> *q*) & p) -> *q*. The second '->' communicates that the 1/0 assignments to the formula to the left of it and the formula to the right of it are members of a set of sets of 1/0 assignments three of which assign that '->' 1 while the remaining set assigns it 0. If the left-hand formula is assigned 0, any 1/0 assignment to the right-hand side makes this set of assignments a member of a set that assigns 1 to '->'. The set of sets of 1/0 assignments for which we use '->' does not have to be what it is; it does not even have to be a set of binary sets. But if that set is what it now is and assigning 0 to the left-hand formula does not assign 1 to the second '->', that set is also not what it is. So, *modus ponens* must be assigned 1 whenever the formula to the left of the second '->' is assigned 0.

If the formula to the left of the second '->' is assigned 1, the formulas to the left and right of '&' must be assigned 1, because of the 1/0-table for which we happen to use '&'. The formula to the right of '&', *p,* is assigned 1 in only one of the sets of assignments that assigns 1 to the formula to the left of '&', (*p* -> *q*), because of the 1/0-table for which we happen to use '->': the set in which *q* is assigned 1. Hence, just as the meaning of 'red' is distinct only by an LGO from that of 'color', assuming that the left-hand side of the second -> is assigned 1 is distinct only by linguistically generated relations of set membership from assuming that the right-hand side is assigned 1. So, the assignment of 1 to *modus ponens* is necessary since assigning either 0 or 1 to the left-hand formula of the second -> is distinct only by linguistically generated relations of set membership from sets that assign 1 to -

>. If assigning the left-hand formula either 1 or 0 does not constitute part of a 1/0-table entry in which '->' is assigned 1, the sets of sets of component 1/0 assignments for which we use '->' and '&' are and are not what they are.

This analysis has not been a proof that, but a causal explanation of why, what a formula expresses is necessarily assigned 1 or 0, namely, the lack of any difference other than a linguistically generated one between what is expressed by a compound formula and particular sets of 1/0 assignments to its atomic formulas. And this analysis has also explained how we know the necessary 1/0 value of formulas using binary operators. The necessary is sometimes defined as a formula that is assigned 1, or whose opposite is assigned 0, on all possible 1/0 assignments to its component formulas. In effect, I am explaining why, and how we can know that, all possible component 1/0 assignments assign 1 or 0 to a formula: because the opposite would require that something is and is not what it is. For each set of component 1/0 assignments, a 1/0 assignment to the compound formula can be contradictory because what each set of component 1/0 assignments is is distinct only by a linguistically generated relation from a set in which the compound formula is assigned 1 or 0, respectively. Since the difference between each set of component 1/0 assignments and a set that assigns 1 or 0 to a truth-functional operator is nothing more than the linguistically generated relation of being a member of the disjunctive set of 1/0 assignments for which we use the operator, if a set of component 1/0 assignments does not assign 1 or 0 to the compound formula, either the set of all sets of 1/0 assignments to the components, or the set of sets of 1/0 assignments for which we use an operator, is not what it is.

One more problem: Red is a member of the set described by 'things employed in traffic control'. That set is linguistically generated since the control in question is the communication of commands. So why isn't 'Red is something employed in traffic control' necessarily true, since what is expressed by 'red' and by 'things employed in traffic control' differ only by an LGO, the very LGO for which the latter is expression is used? What red is does not require it to be a member of that set just as what assigning-1-to-each-of-*p*-and-*q* is does not require that assigning to be a member of the set of sets that assigns 1 to *p* -> *q*. But while that for which we use '->' does make necessary the identity of assigning 1 to *p* and *q* with one of the sets of 1/0 assignments that assigns 1 to *p* -> *q*, that for which we use 'things employed in traffic control' does not make necessary the identity of red with a member of the latter set. Why not, if the 'differs only by an LGO' account of necessary truth is correct?

The LGO account of necessary truth is based on the fact that both necessary and contingent truths require the diverse articulation of something that is not diverse prelinguistically. Where can the diverse articulation come from? What means can we use to cause that diverse articulation? . . . To the extent that the diverse articulation does not come solely from LGOs, by hypothesis it comes from the fact that the means of articulation include the mention of things that are more than linguistically diverse, things whose diversity is prelinguistic with respect to the specific linguistic relations of articulation in question. We can articulate an identical solitary individual as 'the 43rd president of the United States', 'a son of George and Barbara Bush, 'the husband of Laura Bush, 'a former governor of Texas', and so on. So the diverse linguistic expression of one prelinguistic individual can be accomplished by mentioning more than one prelinguistically distinct thing. And since language must begin at the level where 'prelinguistic' must include things existing independently of any kind of cognitional relation, the original prelinguistic diversities language makes use of must be real diversities, diversities that have a status of being, unlike LGOs, more than merely imagined or conceived.

The diverse articulation of the same thing by 'red' and 'something used in traffic control' is the latter kind of diverse articulation; it makes use of the expression in language of things really distinct from one another, what red is, what traffic is, etc. That for which we use 'employed', 'traffic' and 'control' need not be something linguistically generated. Each of these words can describe a really existing, as opposed to a merely conceived or imagined, thing, event, action, state of affairs, quality, or whatever. Given what 'employed' and 'control' are used for, they can be part of a description of an LGO, but primarily they *must* (by the causal necessity shown by the argument against private language) describe real existents. For LGOs come into apprehension only as a result of acts that are more than mere objects of apprehension; they come into apprehension only as a result of acts that really exist. And such acts are among the acts for which we use 'employed' and 'control'.

Each of what 'employed', 'traffic' and 'control' are used for can be distinct from what is expressed by 'red' really, as opposed to just by linguistically generated features. And since the former words are not used solely for LGOs, when we concatenate them into an expression like 'employed in traffic control', the difference between descriptions, like 'red', of individual set members otherwise than as members of the set thus described and descriptions of them as members of that set need not consist solely of the LGO, membership in this set, itself. For the possibility of there being a difference that is more than a merely linguistically generated difference has not been eliminated by the means used to bring the LGO, employed in traffic control, into apprehension. The nature of those means does not require them to produce that effect. The means by which we establish this linguistically generated set can be what they are without that for which we use 'red' being distinct from that for which we use 'something employed in traffic control' only by an LGO.

For the same reason, we could add and subtract members from the set of things employed in traffic control, without any change in what we understand when we understand how 'employed in traffic control' is used. But if we change that for which we use '->' by changing a set of component 1/0 assignments from being a set that assigns '->' 1 to a set that assigns it 0, or vice versa, that for which we use '->' no longer is what it was. So there would be no contradiction if what is expressed by 'red' was not also expressed by 'employed in traffic control'. But what is expressed by 'assigning 1 to *p* and *q*' could fail to be the same as something expressed by 'a set of component 1/0 assignments that assigns *p* -> *q* 1' if and only if the set of sets for which we now use '->' is and is not what it is.

This difference with respect to logical necessity comes from causal necessity. The diverse natures of the causes by which we establish these different LGOs require them to produce effects certain of whose properties are different. Since what that for which we use 'employed', 'traffic' and 'control', are, on the one hand, and what that for which we use 'red' is, on the other, are distinct as really existing behaviors and qualities, only a real connection of causal production or dependence, not a merely linguistically generated relation, could also cause the denial that red is a member of the set of things employed in traffic control to require that something, which is either a cause, an effect, both be and not be what it is. But if we diversely articulate something by mentioning things that are really distinct, how can the prelinguistic identity of the diversely articulated be epistemically necessary? Only a cognition of such a real causal connection between that for which we use these expressions could produce the effect of our knowing a truth to be necessary. And didn't Hume show that since cause and effect are distinct *by definition*, denial of one cannot be evidence for denying the other? But there is a knowable necessary connection between a change newly occurring to something and the distinct reality to which it occurs: A change occurring to something could exist without that to which it occurs existing only if *a change occurring to something is at the same time not a change occurring to something*; so a relation of does-not-exist-without-something-really-distinct-from-itself is part and parcel of *what a change newly occurring to something* is.

On the other hand, if the causes by which we diversely objectify sets consist solely of LGOs, that is, if the description diversifying sets uses words for LGOs only, can there be room for a difference that is more than a linguistically generated difference? In fact there can, for a reason we have already seen in the case of the LGO, negation. The difference between the descriptions 'F' and '~F' is that in the second 'F' is concatenated with a sign used for what is nothing more than an LGO. Does it follow that the members of the two sets so established can differ with respect to nothing more than an LGO and not with respect to anything prelinguistic? No, given what the LGO, negation, happens to be, the exact opposite follows. The paradox is resolved by the fact that the initial relata of linguistically generated relations, the initial objects that are the terms to which a linguistically generated relation like negation applies, must include prelinguistic objects, since those are the objects concerning which language comes into existence in the first place. So words for the LGO negation can be used precisely to express the negation of a prelinguistic sameness.

But if so, when a description establishing a set consists solely of expressions for LGOs, how can we know whether or not being a member of the set is so defined can differ by no more than an LGO from something otherwise described? Only by understanding what the particular LGO or LGOs in question happen to be. It is possible to doubt, if only perversely, that the conditions from which the impossibility of not knowing 'Red is a color' ever occur, since no words in that sentence mention LGOs. But we can fail to know the necessity of

(1) $(p \rightarrow q) \rightarrow (\sim q \rightarrow \sim p)$

only if we do not know that the 1/0-tables for '->' and '~' are what they are. And it would be inexplicable if we could not know that: When we know what a particular 1/0-table is, we are knowing LGOs, sets of sets, that are our own conscious construct.

And it would be inexplicable if more were needed to grasp that necessity than an understanding of what the 1/0-tables we consciously construct for '->' and '~' happen to be. The way we recognize that a truth is necessary must be merely by understanding that for which its words happen to be used, not by a recognition of some marker that would be distinct from a recognition of how those words are used. There couldn't be any magic marker telling which expressions for LGOs leave no room for differences other than those resulting solely from the ways those expressions are used. If there were, its epistemic causal function would be that of a criterion to be applied for knowing that a truth is necessary, which cannot be how we know necessary truth. And here the recognition of the way expressions are used that causes a grasp of necessary truth amounts to an understanding of what a particular linguistically generated relation happens to be. So knowledge of necessities like that of (1) could not require a criterion by which we would recognize that each 1/0-table is the LGO, and recognize the set of sets of LGOs, that it is.

If we cannot grasp that that for which we intend to use an operator is an LGO that we happen to have consciously constructed for that very purpose, we cannot grasp anything needed for recognizing computational correctness, including the empirically verifiable truth that a consciously taken step in a computational proof satisfies rules that we have consciously chosen. Consequently, if we did not have the ability to grasp necessities caused by sets of sets of 1/0 assignments being what they are, we could not go anywhere in modern logic. So if our recognition that certain LGOs happen to be what they are could not cause the grasp of necessary truth, we could not even be having this discussion, which presupposes that we in fact have gone someplace (to say the very least) in modern logic.

In sum: When that for which we use certain expressions differ only by certain LGOs, it can happen that a sentence cannot fail to be true, on pain of contradiction. And it can happen that, when we understand that for which certain of those expressions are used, we can fail to understand the LGOs that make the corresponding truths necessary only on pain of contradicting the contingently true hypothesis without which we could not even be having this discussion, that we happen to understand that for which those words and sentences are used.[53]

Notes

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Simon

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Notes

1. This limitation should, of course, be assumed by the readers of any piece of philosophy. But after 2500 years we philosophers should know better than to assume that readers will do unto us differently from our way of doing unto our fellow philosophers. We certainly intend no injustice to our colleagues, but if we weren't too involved with our own questions to abstract from them completely when reading about different though related questions, we wouldn't be practicing philosophers — unless we're doing it for all the money and fame.

2. Though they must be related somehow, they are distinct questions. As with any set of related philosophical questions, we must understand their distinction in order to understand the relations between them. Those relations may not be what they seem to be. See n. 1.

3. For an example of such pragmatism, see Susan Haack, *Deviant Logic, Fuzzy Logic: Beyond the Formalism* (Chicago: University of Chicago Press, 1996). Haack modifies her view in *Evidence and Inquiry* (Oxford: Blackwell, 1995).

4. 'Logic, Philosophy of', the Routledge Encyclopedia of Philosophy.

5. Pragmatism, p. 10. For Putnam's discussion of Tarski, see Representation and Reality,

Chapter 4, and see 'Does the Disquotational Theory of Truth Solve All Philosophical Problems' in Words and Life.

6. *Representation and Reality*, (Cambridge, MA: Massachusetts Institute of Technology Press, 19??).

7. Saul A. Kripke, *Naming and Necessity* (Cambridge, MA: Harvard University Press, 1990) 19, n.18.

8. Likewise, Einstein had an urgent motivation for using tensor calculus as a tool in general relativity: he couldn't get along without it. Still, that didn't make the tool belong to the same epistemological type as its user, physics. As one of the mathematicians who assisted Einstein made sure to point out, all the new *physics* was contributed by Einstein, not by him. See Abraham Pais, *Subtle is the Lord*, p. ??.

9. Hilary Putnam, *Representation and Reality* (Cambridge, MA: MIT Press, 1988) 117.

10. By 'sociological' philosophic purposes, I mean the purpose of achieving the kind of long-term and cumulatively growing consensus among 'experts', sociologically defined, that experts so defined achieve in other fields.

11. The ECQ example differs from others that illustrate this point, such as knowing that a play in bridge obeys the rules, in the crucial respect that here the rules *are*, or are chosen to reflect something that *is*, logically necessary.

12. This use of 'relation' is non-technical, as opposed to its use for multi-place predicates in Fregean-style, but not all styles, of computational methods in logic. But it can be considered technical with respect to philosophical questions concerning relations. One logician who kindly reviewed an earlier draft of this work balked at calling negation a relation, since it is not a predicate and much less a multi-place predicate. This is a clear example of the power

of methodological imperialism: 'If it is not what my method calls a 'relation' or is not handled the way my method handles what it calls 'relations', it is not a relation.' Prior to Frege, no one could have dreamed of denying that negation was a relation because it was not a predicate and much less a multi-place predicate (a syntactic category which, to my knowledge, did not exist until Frege; for an alternative way of handling relations computationally, see Fred Sommers, *The Logic of Natural Language*). So my friend's reaction was like saying 'If you're not going to play with my ball, I'm not going to play.' Fine, but that does not prevent the rest of us from playing with our ball. His reaction, however, was also a good example of why we must examine the epistemic preconditions of computational methods before making philosophical claims based on them. 'Not' means different-from something, which something is something *else*, by hypothesis. So 'not' always at least implicitly accompanies or is accompanied by (or is associated with, applied to, etc.) some additional intelligible value, Y, as in 'not Y'. Y is therefore the *relatum* of the intelligible value, different-from; in 'not Y', Y is the term of the relation, other-than. ('Not' leaves us with the question 'Not what' just as 'next to' leaves us with the question 'next to what', and for the same reason: next-to is the kind of intelligible value we call a 'relation'.) And 'not Y' implies a reference to a third relatum, something that is different from or considered to be different from Y, some X that is non-Y. That is one way the term 'relation' is precomputationally used, and is a use traditional philosophy has always justifiably had a need for. In that precomputational meaning, negation is a relation. And *how else should you describe negation when you have a need to describe it precomputationally*, as we do here? As an intelligible value additional to negation, Y is distinct from the intelligible value of 'not', although Y may be distinct just as a reduplication of the original use of 'not' for that intelligible value, or as the occurrence of another token of 'not'. The use of 'relation' that (Frege-specific) computational logic has derived from its precomputational use, and that computational logic handles with unprecedented clarity and rigor, is narrower than its precomputational use. And there is absolutely nothing wrong with that. What is wrong is to try to monopolize the philosophical discussion of relation by that computational use, especially when the philosophical discussion concerns the epistemic conditions which the clarity and rigor of that use presuppose.

13. Philosophy of Logic, p. ??.

14. Necessity and our knowledge of it do not depend on the possibility of translating on the basis of behavior. I make reference to behavioral evidence only in reply to the objection that my account implies an illegitimate theory of mental states. Independent arguments below will show that knowledge of necessity occurs and that this knowledge requires no mental states other than those, if any, required for empirical knowledge. The objection applies, if at all, to an after-the-fact psychological account of knowing necessity, not to those arguments. So it is not circular to invoke necessary truths against the indeterminacy of translation. See John C. Cahalan, *Causal Realism: An Essay on Philosophical Method and the Foundations of Knowledge* (Lanham, Maryland: Rowman and Littlefield, 1985) pp. 42-43.

15. The ECQ example differs from others that illustrate this point, such as knowing that a chess move obeys the rules, in the crucial respect that here the rules *are*, or are chosen to reflect something that *is*, logically necessary.

16. Some Renaissance logicians appear already to have seen the argument against ECQ, but perhaps not that all argument fails if we allow contradiction. See E. J. Ashworth, *Language and Logic in the Post-Medieval Period* (Dordrecht: Reidel, 1974) 135.

17. Methods of Logic, 4th ed., p. 177.

18. Methods of Logic, p. 44.

19. A Philosophical Introduction to Set Theory, p. 84.

20. 'Logical Consequence: Model-Theoretic Considerations', section d, *Internet Encyclopedia of Philosophy.*

21. Quine, using Lewis Carroll's 'Achilles-Tortoise Paradox' showed this in 'Truth by Convention', The Ways of Paradox and Other Essays. Note that the necessarily implicit character of the knowledge of inference principles needed for grasping validity is another reason why precomputational knowledge must be non-computational, not acquired by recognizing that steps in a computational procedure are correct according to the rules. The whole point of computational verification is the use of rules sufficiently explicit to permit empirical verification that a step is covered by a rule.

22. Fred Sommers, The Logic of Natural Language

23. Although the differences between precomputational logical knowledge and computational knowledge prevent computational methods from ever adequately modeling *all* our precomputational logical knowledge.

24. And just as computational methods create anomalies for logic, using mathematics as a tool can create anomalies for physics. [??]

25. Peter Rutz, *Zweiwertige und mehrwertige Logik* (Münich: Ehrenwirth, 1973).

26. In brief, my defense of *tertium non datur*, for epistemic purposes, is this: That principle should be read, 'Where something has truth-value, that value is either truth or falsity'. Any argument that some sentence, say, is neither true nor false must show that a condition needed for sentences to have truth-value is missing. Without possessing a complete set of necessary and sufficient conditions for sentences to be true, we can still know that 'This sentence is . . .' (where 'is' does not mean exists) lacks something necessary for sentences to have truth-value. If 'This sentence is . . .' is incapable of having truth-value, how can either 'This sentence is true' or 'This sentence is false' be true? (If 'This sentence is . . .' is incapable of being male or female, neither 'This sentence is male' nor 'This sentence is female' can be true.) Both 'This sentence is *true*' and 'This sentence is *false*' must be false,

and 'This sentence is *neither true nor false*' is true. The premises and conclusions of all critiques of *tertium non datur* lack *any* truth-value unless they are reducible to answers to yes-or-no questions. (In effect, Aristotle correctly argued that sentences need conditions in the world to have truth-value, but he wrongly thought that sentences about the future lack one such condition, the future.)

27. Quine, *Methods of Logic*, 184.

28. The above definition of necessity did not stipulate that only linguistically-generated relations could require the prelinguistic identity of the diversely articulated (Where could that kind of necessity in the second sentence come from?

29. 'Aristotle', in *The Routledge Encyclopedia of Philosophy*.

30. *The Logic of Natural Language*, p. 143 ??.

31. *Representation and Reality* (Cambridge, MA: MIT Press, 1988) 117-118.

32. The red/color example requires only that at least one, but not every, language has a word for color in addition to words for colors. Also, we need not experience more than one type of color to have a word predicable of more than one. Most of us believe in extraterrestrial persons, whether aliens or angels, yet have met only human persons.

33.

Which does require objects like red and color have a status other than being objects of perception (in a crucial sense they don't, but in a crucial sense they do, though that is another matter that itself requires further exhausting work of putting words together in the right ways; see John C. Cahalan, 'Wittgenstein as a Gateway . . .'.) Even so, they are extracognitional in the sense pre-conceptual, that is, pre- the level of cognition that enables us to articulate them in language.

34. 'Wittgenstein as a Gateway to Analytical Thomism' in *Analytical Thomism: Traditions in*

Dialogue, ed. by Matthew Pugh and Craig Patterson (Ashgate: 19??).

35. Nor is the theory of LGOs just a novelty created to explain logical necessity while avoiding these problems. The history of this theory begins with the medieval 'second intention,' which is a species of the medieval 'being of reason, though it develops far beyond that. But as far as I know, nowhere else in that history is the potential of LGOs for explaining logically necessary truth noticed.

36. Nor is the theory of LGOs just a novelty created to explain logical necessity while avoiding these problems. The history of this theory begins with the medieval 'second intention', though it develops far beyond that; see Appendix ??. And as far as I know, nowhere else in that history is the potential of LGOs for explaining logically necessary truth noticed.

37. None of the problems traditionally associated with explaining negative truth should be relevant to the purposes for which I will use this analysis of truth. So for simplicity I will the present the analysis in terms of affirmative truth.

38. Assuming here, for the sake of argument, that prelinguistically there can be no necessary relations between distinct features. There are, but that necessity is causal, not logical, by the definition pertinent to epistemic causal necessity in Hume; see the Introduction.

39. See Yves R. Simon, "The Conformity of Knowledge with the Real," ed. John C. Cahalan, at *Resources for Modern Aristotelian Philosophy.* Visit www.foraristotelians.info*http://www.foraristotelians.info;* at the home page click on "Yves Simon on Thing and Object."

40. For a defense of this claim, see Cahalan, *Causal Realism*, 252-256.

41. W. V. Quine, *The Ways of Paradox and Other Essays* (New York: Random House, 1966) 122-123.

42. Quine's phrase, see 'Truth by Convention', in ??

43. Ibid., 106.

44.. Nelson Goodman, *Fact, Fiction, and Forecast*, 4th ed. (Cambridge, MA: Harvard University Press, 1983) 31, n. 1.

45. If you wonder why we should call negation an LGO, consider that the chances are better for finding hobbits running around the back yard than negations.

46. See Causal Realism, Chapter 11.

47. Which does require objects like red and color have a status other than being objects of perception (in a crucial sense they don't, but in a crucial sense they do, though that is another matter that itself requires further exhausting work of putting words together in the right ways; see John C. Cahalan, 'Wittgenstein as a Gateway . . .'.) Even so, they are extracognitional in the sense pre-conceptual, that is, pre- the level of cognition that enables us to articulate them in language.

48.

Computationally definitions for an operator corresponding to ordinary negation can be derived from prior definitions of, for example, operators for implication or even double negation. But that is just further evidence of the distinction between computational knowledge of logical principles and the noncomputational knowledge that computational knowledge presupposes. Although computational systems can make the definition of other signs more primitive than the definition of signs corresponding to ordinary negation, negation is more primitive *epistemically*. Knowledge of the necessary truth of the inference principles required to know that steps in a computational proof satisfy the rules is knowledge that it would be contradictory if those principles were not true; some LGO would both be and not be what it is. And the LGO that causes our knowledge of the principle of noncontradiction is ordinary negation. So while the wff corresponding to the principle of noncontradiction is computationally on a par with other wffs in the propositional calculus, prior knowledge of the principle of noncontradiction's necessity is required for knowing all the proofs in the propositional calculus. Also, negation is a more fundamental epistemic operation on an atomic statement than any operation that forms a molecular statement. So the ability to truth-functionally define negation and the other operators in terms of 'double negation' clearly illustrates the incommensurability between knowledge of computational correctness and the logical knowledge it presupposes. And precomputational principles of noncontradiction and exclusion of the middle simply expresses the intended effect of negation on the negated, and by hypothesis the *intended* effect of negation is the only one relevant here, since we are talking about that for which we purposefully use negation signs. Of course, principles of noncontradiction and excluded middle also make use of signs for conjunction and alternation, respectively. But it is likewise the intended use of those signs that is relevant. To fail to achieve the intention would be either to change the subject or be mistaken in only the lexicological sense.

49.. Richard Routley, Val Plumwood, Robert K. Meyer, Ross T. Brady, *Relevant Logics and Their Rivals*, v. I (Atascadaro, CA: Ridgeview, 1982) 137; other operators are determinables also.
50. 'Two Dogmas of Empiricism' in *From a Logical Point of View*, p. ??

51.. I once thought it was so unfortunate that Shakespeare had to work under the limitation of writing in iambic pentameter. Think what he could have created if he had been free to use language anyway he wanted. But no; it was because of that limitation, that his language is as magnificent as it is. Overcoming that limitation was precisely what called for such creative brilliance. Likewise, God and angels do not have to use cognitional inventions like negative numbers, and much less square roots of negative numbers; only much more limited and piecemeal intellects do. But it is due to those limitations that we must exercise that kind of inventiveness. We need tricks (like LGOs and other cognitional constructs) to accomplish what God and angels do effortlessly. So our misery is our grandeur. Our misery is the occasion for our most grand displays of cleverness; that cleverness wouldn't be needed were it not for our limitations. Contrary to Wittgenstein, necessity (naturally inevitable restriction) is the mother of invention.

52.. Notice that the necessary truth of inference principles, as well as the necessary validity of inferences, results from a direct extension of the identity relations that ground truth. For example, the necessity of (b) and (B) derives from overlapping identities between things for which their minor, middle and major terms are used. The necessity of (a) and (A) derives from the identities of their major premises' antecedent and consequent, respectively, with the minor premise and conclusion.

53.. I am grateful for the kind help of Hilary Putnam, Alfred Fredoso, Charles J. Kelly, Robert Nozick, James O'Rourke, Michael Pakaluk and Linda Zagzebski and Peter Bernardin.